CONTENTS

Findings from the New Zealand Numeracy Development Projects 2006

Foreword

The Numeracy Development Projects (NDP) began with a pilot in 2000 and since then have expanded to involve almost all of the primary schools in New Zealand. This is the third compendium of papers detailing research into a range of aspects of the NDP. The twelve papers here have been written by academics, independent consultants, and Ministry of Education personnel who are in some way involved in the local mathematics education scene, if not in the NDP itself. The papers in this compendium are arranged under the headings of Student Achievement, Lead Teachers and Sustaining Numeracy in Schools, and Professional Practice.

Student Achievement

"Do They Continue to Improve? Tracking the Progress of a Cohort of Longitudinal Students" (p. 8) by Tagg and Thomas continues a series of papers and reports that go back to 2002. In their research, the performance of students from schools that have completed the professional development stage of the NDP has been considered on two measures. The first of these is the data collected nationally on the strategy domains of the Number Framework and the second is on items for which there are normed scores available. In almost all instances, students in longitudinal schools outperformed those in the comparison groups.

The current paper considers results from 26 schools that have completed the professional development stage of the NDP. The results of previous years are sustained with the present group. Further, students in these schools continue to build on the gains they have achieved. So, for example, year 6 students at schools that have been involved with the NDP over a period of years have a lower proportion of students still using counting strategies and a higher proportion able to use partitioning strategies. In addition, the findings here support anecdotal data that suggests that students' increase in number ability has carried across to other strands, with year 6 longitudinal students scoring 5% higher on non-numeracy test items than the average for New Zealand students as a whole.

"Patterns of Performance and Progress on the Numeracy Development Projects: Findings from 2006 for Years 5–9 Students" by Young-Loveridge (p. 16) follows others of a similar nature by the same author in 2005 and 2006. The current well-researched work looks at the results of over 37,000 students from years 5 to 9 who had been assessed by their teachers at the beginning and end of 2006. Extensive results have been presented graphically.

Young-Loveridge produces a number of important findings. In no special order, these are: first, the gains made by European, Māori, and Pasifika students were comparable, so the differences between their respective performances are not increasing. Indeed, Pasifika and low-decile students appear to be making small gains on their peers. Secondly, it seems that when teachers concentrate on basic facts, there is an improvement in the strategy domains of the Framework. Thirdly, those students who are persistent counters also have weaknesses in place value, basic facts, and number sequences. Finally, year 6 students in this study do not as yet seem to have reached stages that are comparable with the new draft curriculum (2006). This suggests that further professional development may be required in order to bring students up to the desired level.

Irwin and Britt (p. 33) report on a longitudinal study of algebraic thinking in "The Development of Algebraic Thinking: Results of a Three-year Study". The results for the first two years of the study can be found in the 2005 and 2006 compendia.

Four secondary schools were paired with one of their contributing intermediate schools. Year 8, 9, and 10 students from these schools were given an Algebraic Thinking Test consisting of five compensation questions in each of the four arithmetic operations. Within each of the operations, the items developed from purely arithmetic to purely algebraic.

Analyses of the data for the three years of the study showed:

i. high correlations for individuals' scores on the NDP assessments at the end of year 9 and their scores on the Algebraic Thinking Test

ii. a steady increase in Algebraic Thinking Test scores for all year 9 and 10 students in the one school pair for which sufficient data was available

iii. a significant increase in algebraic thinking ability as determined by a comparison of the means of Algebraic Thinking Test scores of all students who had participated in the study over the three years.

The results for the school pair highlighted under (ii) above are of particular interest. In that pair, only the intermediate school had participated in the NDP. Nevertheless, there are features common to both schools that seem to have been crucial to the ongoing development of algebraic thinking. These are discussed in the paper, which draws attention to the role of working flexibly with numerical operational strategies as a basis for developing skills in algebraic thinking and introductory algebra.

New Zealand is a bicultural society, so it is no surprise that the NDP have both an English and a Māori perspective. The Māori-medium version of the NDP is called Te Poutama Tau. Since 2004, research and evaluation in that part of the NDP has been undertaken by Trinick and Stevenson. Their paper this year, "Te Poutama Tau 2006: Trends and Patterns" (p. 44), looks at the overall progress that students made on the Number Framework, in which areas the students performed well and in which not so well, and how progress in 2006 compared with that in 2004 and 2005.

Analyses over the years have shown that there have been positive gains in most areas of the Framework in Te Poutama Tau schools. Further, where there have been areas of concern and teachers have concentrated on these areas, improved performances have resulted.

As a result of the analysis of the data, Trinick and Stevenson make several recommendations for areas on which to focus in 2007. These are:

i. concentrate on older students who have made minimal gains

ii. concentrate on the teaching of addition and proportion, especially in year 4

iii. determine what influence Te Poutama Tau is having on other strands of the curriculum

iv. continue to investigate the relationship between te reo Māori and mathematics

v. determine how younger students can best be prepared for senior mathematics, especially in algebra.

In "'Who helps me learn mathematics and how?': Māori Children's Perspectives" by Hāwera, Taylor, Young-Loveridge, and Sharma (p. 54), 40 children in kura kaupapa Māori schools were interviewed in te reo Māori to find out their views on their learning of mathematics. Among other questions, they were asked "How do you think your teacher helps you to learn mathematics?", "Are there people at home who help you to learn mathematics?", and "How do you prefer to work most of the time – by yourself or with your friends?"

Most students thought that their teacher helped them by showing them strategies, but their responses indicated that the students felt that very little input was required of them in their own learning. The students did not seem to be involved in significant classroom discussions about central mathematical concepts.

Although most students said that they got help from their friends, many students preferred to work by themselves. Reasons for this included fear of being distracted and having their own progress hampered. There was also a feeling that, somehow, collaboration was cheating.

Of the 40 students, 39 cited a range of people at home who helped them on various aspects of their mathematics learning.

Fractions is the focus of Young-Loveridge, Taylor, Hāwera, and Sharma's paper, "Year 7–8 Students' Solution Strategies for a Task Involving Addition of Unlike Fractions" (p. 67). The task used was the addition of $\frac{3}{4}$ and $\frac{7}{8}$, which had to be extracted from a word problem. This task was undertaken in the presence of an interviewer who wanted to know the students' thoughts about learning mathematics as well as how they achieved their answer in the addition task.

As well as providing an interesting, and perhaps worrying, discussion of the results of the interviews, the paper provides a thorough review of the literature on fractions. In this review, the following key points are raised:

i. fractions are an important areas of mathematics
ii. learning about fractions is difficult for most students
iii. these difficulties impinge on students' learning of other areas of mathematics
iv. teaching understanding of fractions can aid students' learning of algebra
v. fractions involve five sub-constructs: part–whole, ratio, quotient, operator, and measure
vi. there is debate in the literature over whether the teaching of algorithms is a good idea
vii. "adding across" denominators as well as numerators is a common error.

The literature motivates the research of this paper in that it looks into an important area of the curriculum and aims to find out students' understanding of simple addition of fractions.

Young-Loveridge et al. found that just over 13% of the 238 students in their study were able to add the two fractions correctly and explain their method. Roughly half as many students again used a correct method but made an error. On the other hand, just under 30% used the "add across" approach.

As a consequence of this piece of research, and in conjunction with the work of Ward, Thomas, and Tagg (p. 87), Young-Loveridge et al. suggest that it should be a high priority to strengthen teachers' knowledge of fractions both at the pre-service and in-service levels.

Lead Teachers and Sustaining Numeracy in Schools

Over the last two years, with the initial phase of the NDP almost complete and nearly every primary and intermediate school in New Zealand having had the opportunity to take part, the focus is moving onto sustaining and improving the gains already achieved.

Papers on sustainability first appeared in *Findings from the New Zealand Numeracy Development Projects 2005* (Ell and Irwin, and Thomas and Ward); in this 2006 compendium, there are papers by Ward, Thomas, and Tagg (p. 87), Higgins, Sherley, and Tait-McCutcheon (p. 99), and Ell (p. 109).

The paper by Ward, Thomas, and Tagg ("Numeracy Sustainability: Current Initiatives and Future Professional Development Needs" p. 87) reports on data received from lead teachers and facilitators in schools that have been involved in the NDP prior to 2006. An on-line survey was developed that focused on two key research questions. These were:

i. To what extent are the sustainability initiatives meeting the professional learning needs of individual teachers?

ii. What elements of numeracy support are needed to sustain or further develop effective numeracy teaching and learning needs in schools?

All schools involved in the NDP in the years since its inception in 2000 were invited to participate. Approximately 26% of lead teachers and 38% of facilitators responded to the surveys. As the result of lead teacher professional development initiatives in 2006, approximately one-third of lead teachers believed that numeracy practices in their school had strengthened. Further, more than half of the lead teachers felt that their learning needs both as a lead teacher and classroom teacher were either "met" or "fully met". Only 10% described their learning needs as "not addressed".

Both lead teachers and facilitators agreed on a number of aspects, including the need to develop teacher content knowledge, especially in the upper stages of the Framework. But they also had different emphases in other areas. For example, lead teachers supported the provision of quality resources as their top priority for successful sustainability, while facilitators thought that the provision of release time for lead teachers would be a more useful course of action.

The paper by Higgins, Sherley, and Tait-McCutcheon ("Leading a Curriculum Reform from Inside a School", p. 99) asks the question "What domains of knowledge inform leadership actions that shift teacher practice and enhance student outcomes?" The paper uses questionnaires and interviews to seek the views of lead teachers, principals, and teachers in an investigation of the knowledge required of lead teachers in the NDP.

Higgins et al. build on Stein and Nelson's 2003 construct of leadership content knowledge, with four categories emerging from their investigation. These are:

i. knowledge of, and attitudes towards, mathematics

ii. knowledge of students as learners

iii. knowledge of teacher as learners

iv. knowledge of communities as learners.

Surprisingly, the relative importance for lead teachers of three of the four categories varies over the three participant groups. Overall, teachers and principals regarded the first and second categories more highly than did the lead teachers, while lead teachers thought that the fourth category was the most important.

Ell's paper "Keeping Going at Country School: Sustaining Numeracy Project Practices" (p. 109) continues her research on sustainability by concentrating this year on Country School (as opposed to the comparison of City School and Country School that Ell and Irwin undertook for the previous compendium). Interviews and videos were used to provide data from an enthusiastic six-teacher rural school.

Patterns and structures appear to have developed in this school that will enable them to continue to use NDP practices. The school continues to embrace, use, and reflect on NDP approaches and on their students' achievement data. The teachers have progressed in their discourse and practice since last year.

Three key points have arisen from this study. These are:

i. Teachers are beginning to use the principles of the NDP in strands other than number. This suggests that the principles of the NDP have been internalised and may support effective practice across the curriculum.

ii. The recognition of children's needs is leading to more carefully planned instruction. Teachers are eager to choose the right activities, ensure the use of appropriate material, and targeting instruction for "where to next".

iii. The role of the NDP resource books seems to be changing from reliance on them (2005) to their use as a guide (2006).

Overall, the gains accomplished in the NDP by Country School are not just on the achievement tests but also in other measures such as the Progressive Achievement Test.

Professional Practice

Annan's paper, "The Numeracy Development Projects: A Successful Policy–Research–Practice Collaboration" (p. 116), brings a new perspective in that he looks at the NDP from a school improvement perspective. In his paper, he clearly enunciates four improvement principles that underlie the NDP. These are:

i. determining students' number knowledge and strategies

ii. designing lessons appropriate to students' abilities

iii. teaching that makes teachers' and students' thinking explicit

iv. checking lesson outcomes using diagnostic and formative assessments.

Annan summarises these as "developing evidence-informed collaborative inquiry" and notes that they are present in other initiatives outside mathematics. He sees this inquiry as enabling the development of relationships among the range of participants – policy developers, resource developers, publishers, facilitators, teachers, and researchers – that evolved out of the task in which they were engaged. He notes two tiers of collaboration that have developed in the NDP. These are strategic and operational. Those in the former tier led the design and evaluation of the NDP, while those in the latter tier were responsible for implementing the NDP in the classroom. These collaborations are moving towards nationwide involvement.

Although the aim of the NDP is to produce better mathematical achievement by New Zealand students generally, Annan is specifically concerned about solving the underachievement problem of disadvantaged children.

The paper by Ward and Thomas, "What do Teachers Know about Fractions?" (p. 128), discusses a tool that they developed to assess teachers' knowledge of the teaching of fractions. In addition, the paper discusses the trial of that tool with a small group of teachers. The tool, developed collaboratively with teachers and facilitators, was focused on the pedagogical content knowledge that teachers require in order to be effective teachers of fractions. This tool comprised a pen-and-paper task based on teaching and learning scenarios involving fractions and proportional reasoning. A typical question asked the teacher if the student's work in a given scenario was correct, and, after showing the student's explanation of that work, asked what, if any, was the key understanding that needed to be developed by that student.

Ward and Thomas found that the tool was both efficient and effective in differentiating between teachers on the basis of their responses. They also found that teachers were more able to answer content questions about fractions than to describe the key concepts involved in the questions. The teachers had difficulty describing the actions they would take next with students in response to the scenarios. Of most concern is the fact that between 30% and 40% of the teacher respondents were unable to solve problems involving operations with fractions and proportional reasoning.

The authors note that "Further work in this area is required to establish a link between teachers' scores in the assessment and student achievement data." Given the responses by teachers here and the corresponding student results from the paper by Young-Loveridge et al. (p. 67), it would seem that the call for such "further work" is justified.

Home–School Partnership: Numeracy (HSPN) began in 2006 as a pilot programme. It follows a similar programme in literacy and is founded on the two notions of the importance of all people and the value of partnership. The aim of the HSPN is to raise the mathematical achievement of Pasifika and other bilingual students by enhancing family and community involvement in their children's learning and is based on the idea that children's learning is increased when school and home act in partnership.

The HSPN programme involved about six community sessions. The families of all children in the schools involved were invited to attend these sessions, which included principles and pedagogy from the NDP. The sessions were led by lead teachers and selected parents (lead parents), who had attended a number of training workshops to prepare them for leading the community sessions. The paper "Exploratory Study of Home–School Partnership: Numeracy" (p. 139), by Fisher and Neill summarises the findings of a study into that pilot.

A number of factors were identified by Fisher and Neill that were important for the success of the HSPN. Among these were careful selection of the lead parents, ensuring that the community sessions were engaging to parents, and providing mathematical exploration that relates to real life. On the other hand, Fisher and Neill also noted areas in which the programme might be improved, such as providing more opportunities for the community sessions to be in parents' first language, having a succession plan to ensure continuity, and developing more ways to reach the community and get parents to attend.

Conclusion

A number of common themes appear through the papers of this compendium. For instance, the large body of statistics that is being added to each year shows continual progress being made in students' results across the board. This progress can be seen from almost all of the papers in the Student Achievement section. Although some groups still have progress to make, one of the pleasing aspects is that the groups that are behind are not getting further behind each year. Indeed, there are signs that some gaps are decreasing, albeit in a small way. Evidence for this can be seen in Trinick and Stevenson (p. 44) and Young-Loveridge (p. 16).

It is also pleasing to note that teachers are beginning to apply the pedagogy promoted in the NDP to other strands of the curriculum. Ell (p. 109) and Tagg and Thomas (p. 8) both mention this aspect of teachers' work.

Also on the positive side is the fact that students are managing to do well on a variety of standard tests outside of the NDP (PAT, asTTle, TIMSS, and NEMP). Evidence is found for this both in this compendium (Tagg and Thomas, p. 8, and Ell, p. 109) and in the previous compendium (Thomas and Tagg p. 22).

However, there are clearly areas where effort is going to be required in the immediate future. This is especially true for fractions and proportional reasoning, which are acknowledged as matters of general concern in the community as a whole. The work of Young-Loveridge (p. 16), Ward and Thomas (p. 128), and Trinick and Stevenson (p. 44) also exposes them as a problem for both English-medium and Māori-medium schools. It is clear that an emphasis will have to be put on fractions and proportional reasoning in professional development for some time yet.

Nevertheless, despite the areas of concern, there is no doubt that the NDP, supported by a stack of data and a wealth of research, is one of the leading teacher professional development programmes in mathematics in the world.

Professor Derek Holton
Department of Mathematics and Statistics, University of Otago

Do They Continue to Improve?
Tracking the Progress of a Cohort of Longitudinal Students

Andrew Tagg
Maths Technology Ltd.
<andrew@nzmaths.co.nz>

Gill Thomas
Maths Technology Ltd.
<gill@nzmaths.co.nz>

Findings from the Numeracy Development Projects (NDP) have consistently shown that students in participating schools make significant gains on the Number Framework. Since 2002, the NDP Longitudinal Study has shown that students continue to perform well in the years following their school's completion of the professional development, with a greater proportion of students in longitudinal schools reaching higher stages of the strategy domains of the Framework than those in first year NDP schools. This paper reports on an analysis of the performance of a cohort of longitudinal students over the last five years. The findings indicate that these students continue to build on their progress, with year 6 students in longitudinal schools outperforming those in first year NDP schools by an average of almost half a stage across the strategy domains.

Background

The Numeracy Development Project (NDP)

In each year since the implementation of the pilot project in 2000, research reports have been written describing the progress made by students in schools participating in the professional development phase of the NDP (for example, Thomas, Tagg, & Ward, 2003; Thomas & Tagg, 2003, 2004, 2005, 2006; Young-Loveridge, 2004, 2005, 2006). The size of the gains made has been analysed by comparing mean stages, comparing changes to the proportions of students at each stage of each domain, and examining proportions of students making gains. Comparisons have also been made between the performance of students at the end of their school's professional development year and that of pre-professional-development students from the next year level (Thomas & Tagg, 2003, 2004; Young-Loveridge, 2005). These research reports have consistently shown that students in the professional development phase of the NDP make gains in their ability to operate with numbers as measured by the Number Framework, and that these gains are larger than those expected in a non-NDP context (Bobis, Clarke, Clarke, Thomas, Wright, Young-Loveridge, & Gould, 2005).

The Longitudinal Study

Since 2002, the NDP Longitudinal Study has tracked the progress of students in schools in the years following their professional development year (Thomas, Tagg, & Ward, 2003; Thomas & Tagg, 2004, 2005, 2006). The Longitudinal Study has tracked the performance of students in two ways: comparing the performance of students in these schools on the strategy domains of the Framework with that of students nationally, and testing students from longitudinal schools using items on which normed scores for New Zealand students are available for comparison. Findings from the Longitudinal Study have consistently shown that a greater proportion of students from longitudinal schools achieve higher stages on the Framework than students from schools in their professional development year. Additionally, with the exception of year 6 students in 2005, longitudinal students in years 4 to 6 have achieved 4–9% higher on tests comprising items from all areas of mathematics when compared to the New Zealand students on the assessments from which the items were sourced.

This paper analyses the performance over time of the cohort of students from the longitudinal schools who were in year 6 in 2006. It also compares the performance of year 6 students in longitudinal schools in each of the five years of the study.

Method

Participants / Procedure

The Longitudinal Study began in 2002 with the participation of 20 schools that first implemented the NDP in either 2000 or 2001. While there have been a number of changes to the schools participating each year since the start of the study, every effort has been made to keep the demographic profile of the sample consistent. Each year, new schools to replace those that have withdrawn are randomly selected from a list of schools that completed NDP training in the previous years. The list is stratified by decile to ensure that those selected and invited to participate in the Longitudinal Study closely approximate the national sample and that there are similar numbers of students in years 1–8. Of the 26 schools involved in 2006, 11 schools have participated in the Longitudinal Study since its inception, three were added in 2004, seven in 2005, and five in 2006. The five schools new to the study in 2006 were two high-decile schools, one medium-decile school, and two low-decile schools. The decile profile for 2006 was skewed by one of the new low-decile schools not returning numeracy results and by changes to decile ratings that caused two of the existing low-decile schools to be re-categorised as medium-decile schools. Table 1 gives the percentages of students in each decile band for which numeracy data was received for each year of the Longitudinal Study.

Table 1
Decile Band of Longitudinal Students

	Low decile (1–3)	Medium decile (4–7)	High decile (8–10)	Total
2002	33%	37%	30%	2 362
2003	39%	31%	30%	3 416
2004	39%	30%	31%	6 099
2005	30%	43%	27%	6 826
2006	15%	40%	46%	7 386

Schools participating in the 2006 Longitudinal Study were asked to provide the stages of all their students on each of the additive, multiplicative, and proportional strategy domains of the Framework in the final term of the year. Schools were instructed to collect their students' strategy stages in whatever way was most convenient for them; they were also told that full diagnostic interviews were not required. This data was entered on the online Numeracy Database for analysis.

Year 6 students in longitudinal schools were also asked to complete a written test, made up of a combination of items from the Assessment Resource Banks (NZCER, n.d.) and the Progressive Achievement Test of Mathematics (NZCER, 1993, 1994) and including items from all strands of the mathematics curriculum. These students had completed similar tests in year 4 and year 5 made up of items from the Trends in International Mathematics and Science Study (TIMSS) 1995 and TIMSS 2003 assessments respectively (Thomas and Tagg, 2005, 2006).

This paper focuses on the results of students in year 6 in 2006, who will be referred to as the target cohort. The performance of the target cohort is compared with the performance of the same cohort in previous years as well as with that of students in year 6 in longitudinal schools in previous years.

Findings

Performance of the Target Cohort on the Number Framework

Figure 1 shows the mean additive stages of students from the target cohort compared with students in the same year level from schools participating in the NDP professional development nationally from 2002 to 2006. The recently released draft mathematics curriculum (Ministry of Education, 2006) includes achievement objectives in the number strand that relate very closely to the stages of the Framework. The curriculum describes the counting stages (1–4) as being at level 1, early additive (stage 5) at level 2, and advanced additive (stage 6) at level 3. Because the levels of the curriculum are seen as roughly equivalent to two years of schooling, it is possible to identify the stages of the Framework at which the curriculum indicates students should be achieving. These stages are indicated by crosses on the graph to show the expected achievement of year 2, year 4, and year 6 students. The third line on the graph indicates the performance of students who have not been in NDP classes. Data for this line has been obtained by using the start-of-year data for the next year level from the NDP, so that, for example, year 3 initial data for 2002 is compared with year 2 final data. This means that the students represented by this line are, on average, six months older than those represented by the other two lines. The number of students for whom results were available for the longitudinal cohort ranged from 677 in 2002 to 1 050 in 2006, while there were over 6 500 students included in each year for each of the two national groups.

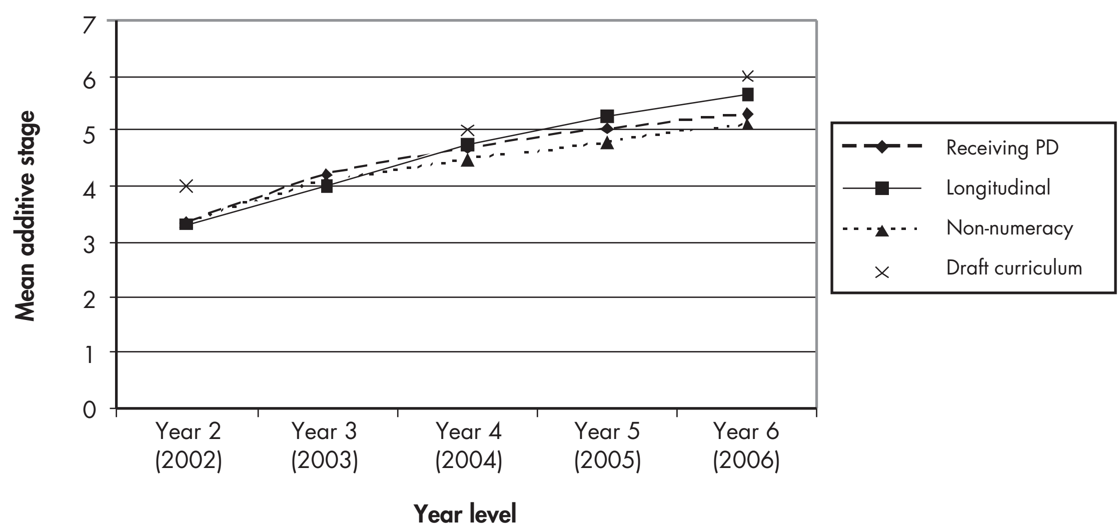


Figure 1: Mean additive stages

The performance of the target cohort as year 2 students in 2002 was very similar to that of year 2 students nationally (mean stage of 3.3 for both groups). This is as expected, considering that the schools had only recently completed the NDP professional development. In 2003, the performance of the target cohort in year 3 was poorer than that of year 3 students nationally (mean stage of 4.0 compared to 4.2). This may be explained by the increased coverage of other strands of the mathematics curriculum in the year following the first implementation of the NDP. By year 4, the two groups performed similarly, and in year 5 (5.3 compared to 5.0) and year 6 (5.7 compared to 5.3), students from the target cohort outperformed year 6 students from schools in their first year in the NDP. The mean stage of year 2, 4, and 6 students on the additive domain is close to that indicated by the draft curriculum. The students from the target cohort and students from schools in their first year in the NDP consistently have higher mean stages than students without exposure to numeracy practices, despite the six-month age deficit.

Figures 2 and 3 show the mean multiplicative and proportional stages of the target cohort from 2002 to 2006. The pattern of performance illustrated is similar to that shown on the additive domain, with the mean stage of the longitudinal students nearly half a stage higher than that of students from schools in their professional development year by the end of year 6. The performance of year 3 students in the target cohort is consistently lower than that of year 3 students in schools undertaking NDP training. This may reflect the almost exclusive focus on numeracy in the classroom mathematics programme during NDP training. The mean multiplicative and proportional stages of year 4 and 6 students are again close to those indicated by the draft mathematics curriculum and higher than for students whose schools have not yet participated in the NDP. The fact that the year 2 mean stages on these domains are lower than that indicated by the curriculum is largely due to the fact that students rated below stage 4 on the additive domain are not tested on the multiplicative or proportional domains. These students are rated as a zero for the purposes of calculating mean stages.

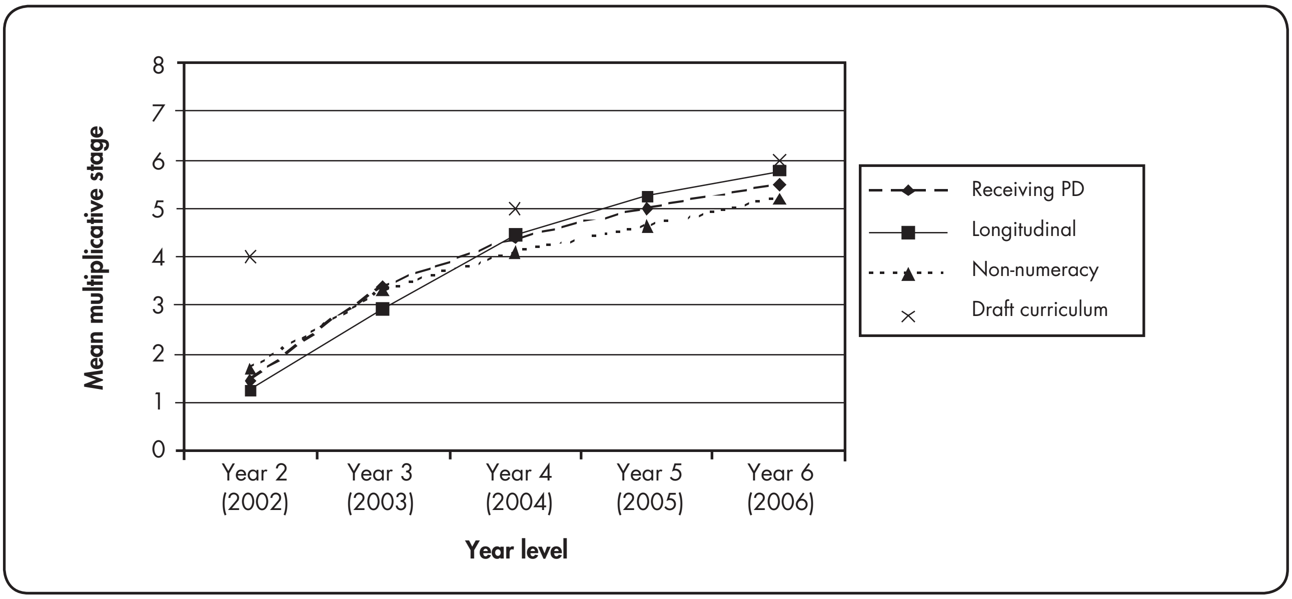


Figure 2: Mean multiplicative stages

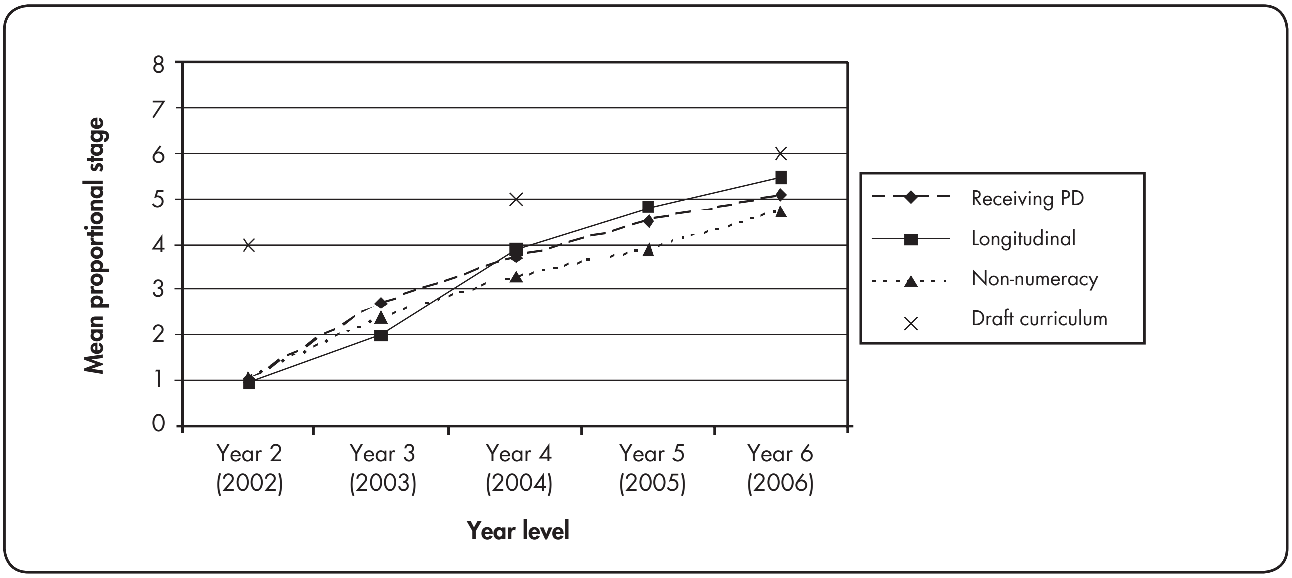


Figure 3: Mean proportional stages

Performance on the Number Framework of Year 6 Students in Longitudinal Schools 2002–2006

Figure 4 shows the proportion of year 6 longitudinal students rated as at least advanced additive (stage 6) on each of the three strategy domains for the five years of the Longitudinal Study. A clear trend can be seen, with at least 20% more students reaching the top stages of each domain in 2006 than in 2002. This graph indicates that, as schools continue to implement numeracy practices over time, an increasing proportion of their students finish year 6 at the advanced additive stage of the Framework.

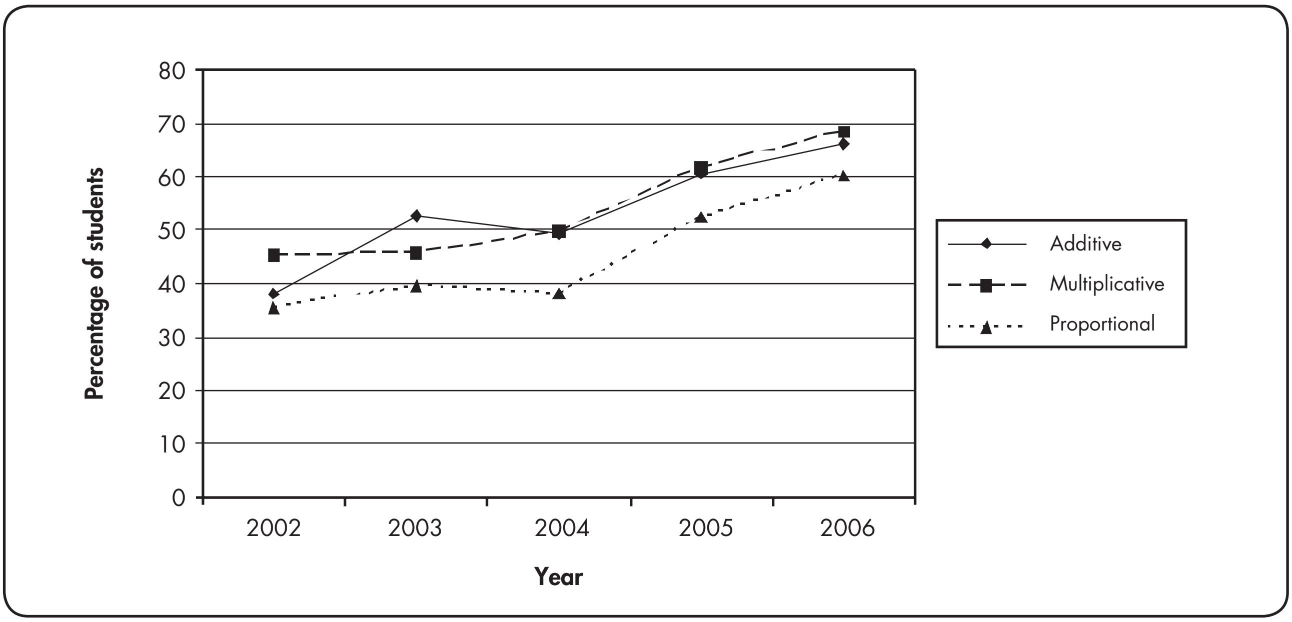
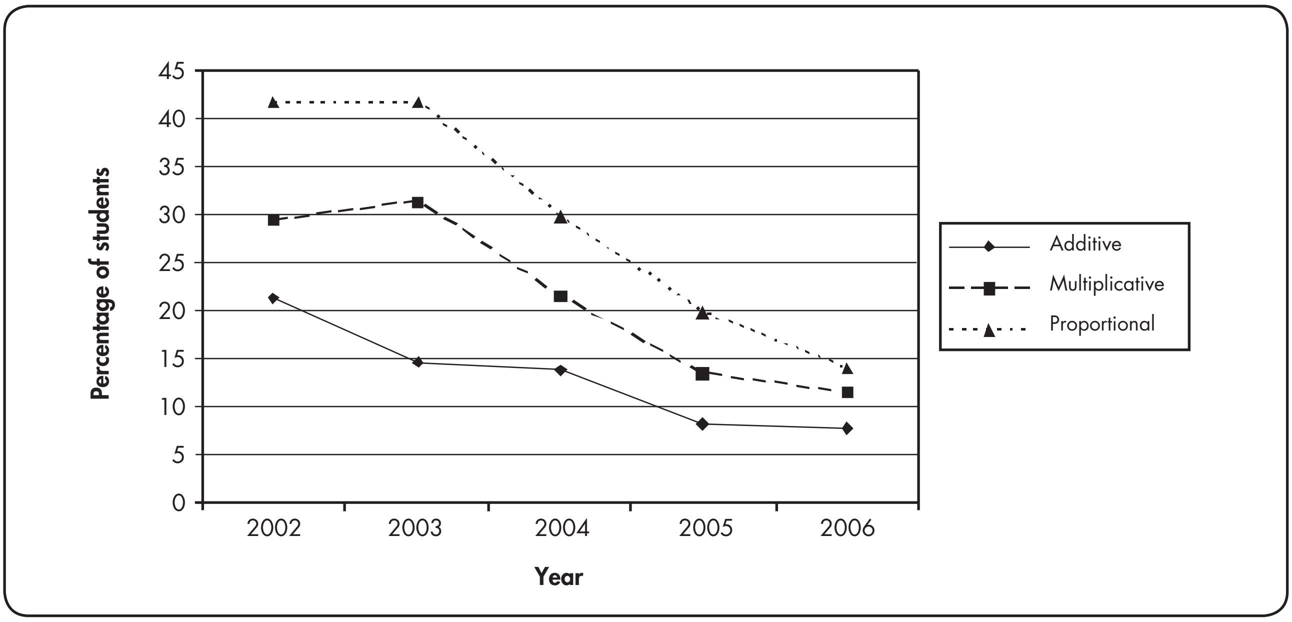


Figure 4: Percentage of year 6 longitudinal students rated as at least advanced additive on the strategy domains

Between 2002 and 2006, the percentage of year 6 students in longitudinal schools reaching at least stage 6 increased from 38% to 65% on the additive domain, from 45% to 67% on the multiplicative domain, and from 36% to 59% on the proportional domain. As a point of comparison, in 2005, 41% of year 6 students from schools in their professional development year reached stage 6 or higher on the additive domain, 56% on the multiplicative domain, and 45% on the proportional domain (Young-Loveridge, 2006). This indicates that a greater percentage of year 6 students in longitudinal schools attain at least stage 6 (level 3 of the draft mathematics curriculum) on each domain than do students in schools who are in the professional development year.

"Expectations" in the "Principal Support, Guidelines for the use and reporting of student achievement data" section of the nzmaths website describe year 6 students who are still rated as only able to use counting strategies (stage 4 or below) as "at risk" (Maths Technology Ltd, n.d.). Figure 5 shows the proportions of year 6 students in longitudinal schools rated as stage 4 or below in their end-of-year assessment. By 2006, the percentage of students still at risk on the additive domain had decreased from 21% to 8%, the percentage on the multiplicative domain had decreased from 29% to 12%, and the percentage on the proportional domain had decreased from 42% to 14%. In comparison, the national results for schools in their professional development year in 2005 showed 14% of year 6 students rated as stage 4 or below at the end of the year on the additive domain, 19% on the multiplicative domain, and 25% on the proportional domain (Ell, Higgins, Irwin, Thomas, Trinick, & Young-Loveridge,



2006).

Figure 5: Percentage of year 6 longitudinal students rated below early additive on the strategy domains

Figure 6 shows the mean stages of year 6 students on each of the three strategy domains. The trend is again positive for students in longitudinal schools, with the mean end-of-year stage on the multiplicative and proportional domains increasing by approximately one stage over the five years of the Longitudinal Study. For example, the mean proportional stage for year 6 students has increased from 4.3 in 2002 to 5.6 in 2006. Improvement on the additive domain is more modest, though a "ceiling" effect may apply in this instance because there is no stage 8 on this domain and several of the longitudinal schools continue to use older versions of the NumPA that only extend to stage 6 on the additive domain.

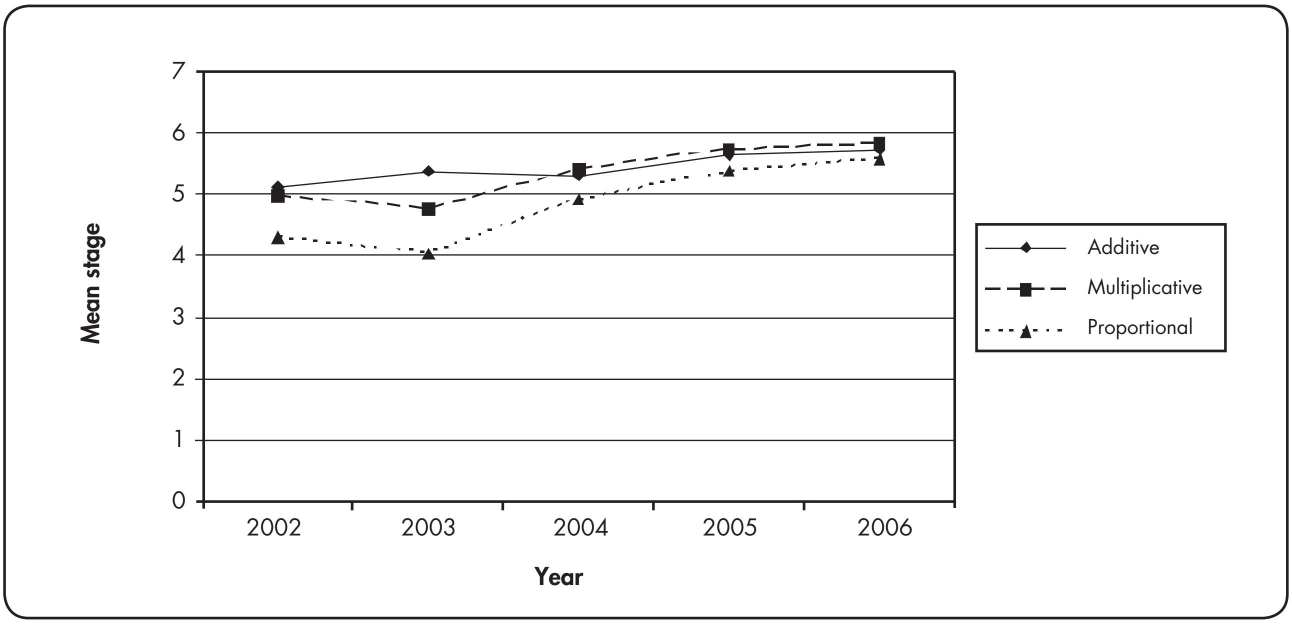


Figure 6: Mean stage of year 6 longitudinal students on the strategy domains

Performance of the Target Cohort on Written Tests

Since 2003, the Longitudinal Study has also measured the performance of students in selected year groups on pen-and-paper tests containing items whose content encompassed all strands of the mathematics curriculum. These tests aimed to determine the impact of the NDP on students' overall performance on mathematics. The items from these tests were sourced from assessments that included a significant sample of New Zealand students prior to the implementation of the NDP. The norms used for all but one of these items were found prior to the implementation of the NDP. Students in the sample cohort completed longitudinal tests in years 4, 5, and 6. Table 2 shows the source of the items in each of the three tests.

Table 2
Source of Items in Longitudinal Tests

	Year 4 (2004)	Year 5 (2005)	Year 6 (2006)
TIMSS 1995	24		
TIMSS 2003		24	
Assessment Resource Banks (ARBs)			13
Progressive Achievement Tests (PATs)			15

Table 3 compares the percentages of items answered correctly by the target cohort with those of students in the original assessments. Items in the tests were related to all strands of the curriculum. The table shows the performance of students on items specifically relating to the NDP practices as well as the performance on items with content not directly related to the NDP. In all cases, the performance of longitudinal students was better than that of students from the source assessments. On the items identified as not being related to the NDP, students from the target cohort consistently gave between 3–5% more correct answers than students in the source assessments. In 2004 and 2006, the longitudinal students performed particularly well on the NDP items, giving approximately 10% more correct answers than the students in the source assessments.

Table 3
Percentages of Items Correct for Longitudinal Students Compared with New Zealand Norms

	NDP		Other		Total	
	Long.	NZ	Long.	NZ	Long.	NZ
Year 4 (2004)	55	45	57	54	56	50
Year 5 (2005)	51	48	57	53	54	50
Year 6 (2006)	58	47	60	55	59	50

These findings support anecdotal comments from teachers in longitudinal schools who have suggested that students' increase in number knowledge has been translated to other strands and that students have also shown an increased enthusiasm for mathematics, which has led to improved performance (Thomas and Tagg, 2004).

Concluding Comment

The findings of the Longitudinal Study indicate that the NDP continue to impact positively on students in the years following their initial implementation. Further to this, it appears that students in longitudinal schools continue to build upon gains, with the end-of-year performance of year 6 longitudinal students improving over the course of the study. Schools that have implemented numeracy practices for an extended period have lower proportions of students by the end year 6 still restricted to counting strategies and higher proportions of students able to use a range of partitioning strategies.

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Patterns of Performance and Progress on the Numeracy Development Projects: Findings from 2006 for Years 5–9 Students

Jenny Young-Loveridge
University of Waikato
<jenny.yl@waikato.ac.nz>

The purpose of this study was to analyse the data from diagnostic interviews conducted with year 5–9 students by teachers who participated in the professional development programme of the Numeracy Development Projects (NDP) in 2006. By the end of a year on the NDP, just under half of the year 6 students were able to use a range of additive strategies to solve addition and subtraction problems (stage 6). Between one-third and one-half of the year 8 students were able to use a range of multiplicative strategies to solve problems with multiplication, division, and fractions (stage 7). These findings raise issues about some of the achievement objectives in the draft New Zealand Curriculum (2006) and the need to provide further intensive support for teachers if the majority of students are to meet these new objectives at the levels stated in the new curriculum document. The proportion of students at the upper stages of the Framework has increased over time, but this coincides with an increase in the proportion of students from high-decile schools taking part in the NDP and correspondingly a decrease in the proportion of students from low-decile schools. The analysis of effect sizes for comparisons between younger students after the NDP with slightly older students before they began the NDP shows that the impact of the NDP was greatest for Pasifika students, who had the largest effect size, on average (0.40). The average effect size for students from low-decile schools was 0.38, while that for Māori students was 0.35, slightly greater than that for European students (0.33). This analysis suggests that when comparisons are made between students within the same subgroup, those who have traditionally had lower levels of achievement (Māori and Pasifika students and those from low-decile schools) seem to benefit the most from participating in the NDP. Previous comparisons, between Māori/Pasifika and European students at identical stages on the Framework initially, showed that European students made the greatest progress in terms of gains in stages on the Framework. Likewise, simple comparisons between these subgroups on initial and final stages on the Framework showed that European students began at higher stages on the Framework and made greater gains than Māori or Pasifika students. Overall, the data suggests that the achievement gap, while not necessarily narrowing as a result of participation in the NDP, is being prevented from becoming larger, and this effect is greatest for Pasifika students and students from low-decile schools. Analysis of students' performance on basic facts and place value suggests that a focus on building students' knowledge of basic facts and an understanding of place value may lead to improved performance on the operational domains of the Framework.

Introduction

The New Zealand Numeracy Development Projects (NDP) have now been underway for more than seven years. Like other educational reform initiatives worldwide, the NDP were set up to improve mathematics teaching and learning at primary and secondary levels (Ministry of Education, 2001). Analysis of the data on students' mathematics achievement gathered by teachers as part of the professional development (PD) programme has been a valuable source of information for shaping PD in subsequent years (see Young-Loveridge, 2005, 2006). This paper reports on the results for the NDP for 2006, focusing particularly on students in years 5–9.

Method

Participants

In 2006, 37,144 year 5–9 students were assessed at the beginning and end of the year in which their teachers participated in one of the PD programmes for the NDP. Almost two-thirds (64.4%) of the cohort was European, while close to one-fifth (18.6%) was Māori. The remainder of the cohort consisted of Pasifika (7.4%), Asian (4.9%), and students from other ethnicities (4.7%). Compared to the national picture, this cohort included disproportionately more students from high-decile (39.3%) and medium-decile (43.7%) schools, and disproportionately fewer students from low-decile (17.0%) schools. (Note: The decile ranking of the school is used as an indicator of socio-economic status, with low-decile schools constituting the lowest 30% in socio-economic status, medium-decile schools the middle 40%, and high-decile schools the highest 30%.) The cohort was balanced in gender, with 51.3% being boys and 48.7% girls. Appendix A (p. 154) shows the composition of each year group in the 2006 cohort, as well as those for the years 5–9 cohorts that participated in the NDP between 2002 and 2006. (Note: Year 9 data is included for just 2005 and 2006 – the years since the Secondary Numeracy Pilot Project 2005 [see Harvey, Higgins, Maguire, Neill, Tagg, & Thomas, 2006].)

Procedure

Students were interviewed individually by their own teachers, using the NumPA (Numeracy Project Assessment, Ministry of Education, 2006) diagnostic assessment near the beginning of the school year (initial), and again near the end of the year (final). Data from these assessments was forwarded to a secure website for later analysis. Only students with both initial and final data were included in the analysis for this paper.

Results and Discussion

Performance of Students Participating in the NDP

The percentages of students in years 5–9 at each stage on the Number Framework at the beginning and end of the school year are presented in Appendix B (p. 155) for all three operational domains (addition-subtraction [additive domain], multiplication-division [multiplicative domain], and proportion-ratio [proportional domain]) and for the knowledge domains of fractions, place value, and basic facts. It was interesting to note that by the end of the school year, just under half (49.2%) of the year 6 students had reached stage 6 (advanced additive) on the additive domain, an expectation currently at level 3 of the draft New Zealand Curriculum (see Ministry of Education, 2006). By the end of year 8, that proportion had increased to just under two-thirds (65.3%), still somewhat short of the substantial majority one would hope to see for an achievement objective at a particular curriculum level. The 2006 cohort (see Appendix A, p. 154) included disproportionately more students from high-decile (39.3% instead of 30%) and medium-decile schools (43.7% instead of 40%) and disproportionately fewer students from low-decile schools (17.2% instead of 30%). Hence the proportions of students at stage 6 and stage 7 may be greater than is typical of a more representative cohort. By the end of year 8, just over two-fifths (41.1%) of the students had reached stage 7 (advanced multiplicative) on the multiplicative domain, an expectation currently at level 4 of the draft curriculum. This rather disappointing result has major implications for secondary schools and the need for students to be multiplicative if they are to succeed with algebra (Lamon, 2007).

It may be that, over time, as more teachers become familiar with the Framework and assessment tools and more students have learned mathematics with the assistance of an NDP-trained teacher for the whole (or at least the majority) of their school careers, the numbers of students at upper stages of the Framework will increase. The results of the longitudinal study show greater proportions of students at these levels in schools that have been using NDP tools and resources for several years, with approximately 70% of year 6 students at stage 6 or higher (see Thomas & Tagg, 2006), which is about 20% more students than at the end of one year of NDP professional development. However, the 47% of year 8 students in the longitudinal study findings at stage 7 (see Thomas & Tagg, 2005) is only slightly more than the 41% found in the present study. It seems likely that the longitudinal results are biased favourably by schools that self-select their continued participation in the longitudinal study on the basis of the success they experience in working with NDP tools and resources.

Looking Back over the Last Five Years

Appendix C (p. 158) shows the proportion of students at different stages on the Framework for the three strategy domains plus the fractions domain at the end of the school year in which their teachers were trained to use NDP tools and resources. The proportion of year 6 students reaching stage 6 on the additive domain has increased steadily from 36.3% in 2002 to 49.2% in 2006, an increase of almost 13%. However, the increase in the proportion of year 8 students reaching stage 7 on the multiplicative domain is only 7.3% (from 33.8% to 41.1% over the same years). The cohort has changed from comprising students from mostly low-decile and medium-decile schools in 2002 (33.0% low-decile and 47.6% medium-decile) to comprising students from mostly medium-decile and high-decile schools in 2006 (43.4% medium-decile and 39.3% high-decile). Hence the increase in proportions of students at the upper stages is likely to be related to these changes in cohort composition.

Differences between Initial and Final Stages on the Framework

Table 1 shows the average stages on the Framework at the beginning and end of the year on the three strategy domains, the differences between subgroups, and the associated effect sizes for comparisons between those subgroups. It is evident from Table 1 that on the additive and multiplicative domains, the students gained just over half a stage on the Framework, on average. On the proportional domain, the gain was closer to a whole stage. There were small differences between subgroups (for example, Pasifika students gained slightly more, on average, than European or Māori students). However, these findings were confounded by the fact that the Framework does not consist of an interval scale. On the additive domain, steps at the lower stages of the Framework are smaller than at upper stages. Hence the average stage gain can appear greater for students who start lower on the Framework. The findings for the multiplicative and proportional domains were further confounded by the fact that, in order to maintain appropriate relativities with stages on the additive domain of the Framework, some steps on the Framework consist of a range of stages (for example, on the multiplicative domain, there is no stage 1, and the first step is stages 2–3 [count from one]; the proportional domain has stage 1 as its first step, but the second step is stages 2–4 [equal sharing]; fractions has no stage 1 but has stages 2–3 [unit fractions not recognised]). For the purpose of calculating means, standard deviations, and effect sizes, the first step on the multiplicative domain and the second step on the proportional domain were allocated a value of 3, resulting in much greater variability in students' stages and contributing to larger pooled variance for the calculation of effect sizes. For these reasons, the means, standard deviations, and effect sizes need to be treated with caution.

Comparing Groups

It is clear from Table 1 that European students started higher on the Framework than students with Māori or Pasifika ancestry (for example, 5.19 compared with 4.88 or 4.66 respectively, on the additive domain in 2006). This difference increased slightly by the end of the school year for Māori, but only on the additive (0.31 to 0.33) and multiplicative (0.39 to 0.43) domains. The effect sizes for these differences were about one-third of a standard deviation for the European to Māori comparison (0.34 and 0.36 on the additive domain initial and final). In 2006, the differences between European and Pasifika students reduced slightly over the school year from the initial to the final assessments. The effect sizes for these differences were just over half a standard deviation for the European-Pasifika comparison (0.57 and 0.53 on the additive domain initial and final), but on all three strategy domains, the effect sizes were smaller for the final assessment than for the initial one. The comparison between students from high- and low-decile schools yielded effect sizes of just over half a standard deviation, and this increased slightly for the final assessments compared to the initial ones.

Table 1

Average Stage on the Framework, Differences between Subgroups and Associated Effect Sizes for Initial and Final Stages on Each of the Three Strategy Domains 2005 & 2006

Domain	European	Māori	Difference European–Māori	Effect Size	Pasifika	Difference European–Pasifika	Effect Size	High Decile	Low Decile	Difference High–Low	Effect Size
2005											
Additive											
Initial	4.99	4.68	*0.31*	**0.34**	4.50	*0.49*	**0.54**	5.04	4.54	*0.50*	**0.54**
Final	5.49	5.19	*0.30*	**0.34**	5.00	*0.48*	**0.54**	5.56	5.06	*0.50*	**0.54**
Gain	*0.50*	*0.51*			*0.50*			*0.52*	*0.53*		
Multiplicative											
Initial	5.18	4.77	*0.41*	**0.36**	4.54	*0.64*	**0.56**	5.24	4.61	*0.63*	**0.55**
Final	5.88	5.44	*0.44*	**0.38**	5.23	*0.65*	**0.57**	5.96	5.31	*0.65*	**0.55**
Gain	*0.70*	*0.67*			*0.69*			*0.72*	*0.70*		
Proportional											
Initial	4.63	4.00	*0.62*	**0.37**	3.59	*1.04*	**0.61**	4.71	3.75	*0.96*	**0.56**
Final	5.58	4.94	*0.65*	**0.43**	4.69	*0.89*	**0.60**	5.68	4.79	*0.89*	**0.58**
Gain	*0.95*	*0.93*			*1.10*			*0.96*	*1.04*		
2006											
Additive											
Initial	5.19	4.88	*0.31*	**0.34**	4.66	*0.53*	**0.57**	5.26	4.75	*0.52*	**0.54**
Final	5.68	5.35	*0.33*	**0.36**	5.19	*0.48*	**0.53**	5.78	5.23	*0.55*	**0.59**
Gain	*0.49*	*0.47*			*0.53*			*0.52*	*0.48*		
Multiplicative											
Initial	5.40	5.01	*0.39*	**0.34**	4.68	*0.71*	**0.62**	5.45	4.84	*0.61*	**0.52**
Final	6.05	5.62	*0.43*	**0.38**	5.40	*0.65*	**0.58**	6.14	5.48	*0.66*	**0.57**
Gain	*0.65*	*0.61*			*0.71*			*0.69*	*0.64*		
Proportional											
Initial	5.00	4.36	*0.64*	**0.38**	3.99	*1.01*	**0.61**	5.05	4.10	*0.95*	**0.56**
Final	5.86	5.25	*0.61*	**0.42**	5.01	*0.85*	**0.59**	5.97	5.03	*0.94*	**0.63**
Gain	*0.86*	*0.89*			*1.02*			*0.92*	*0.93*		

The Impact of the NDP on Students' Performance

The results show that students reached higher levels on the Framework at the end of the year than they had at the beginning of the year. However, it is likely that some of the progress that students made was as a result of "normal" aging rather than because they had participated in the NDP. In a traditional experimental study, the progress (as measured by the difference between pre-test and post-test scores) of the intervention group (those taking part in the project) would be compared with the progress of the control group (those not taking part in the project). There were good reasons why a traditional "control" group was not used in this instance (see Young-Loveridge, 2005). For example, there would have been ethical issues if the opportunity to participate in the NDP had been withheld from some teachers and their students for the purpose of creating a control group. There would also have been logistical problems in training non-participant teachers to assess their students simply for the purpose of comparison with students whose teachers did participate in the NDP. The teachers' assessment of their students was an important dimension of the PD programme for the NDP, so getting outside researchers to interview a control group of students would have introduced a confounding variable that could have been responsible for any differences between intervention and control groups, thus defeating the purpose of taking such a step. However, using students at the next year level up (the adjacent year group) before they participated in the NDP allowed comparisons to be made between students who had experienced a year of the NDP and those who not yet participated (pre-NDP).

For example, year 2 students who were at the end of a year of the NDP were compared with year 3 students who were at the beginning of a year of the NDP. On average, the year 2 students were about one-quarter of a year younger at the final assessment than the year 3 students at their initial assessment, so this provides a conservative measure of "control" for the "intervention" group. Figure 1 shows the comparison for the additive domain between younger students after a year of the NDP and older students before they began on the NDP. It is clear from Figure 1 that there were few differences between younger students after the NDP and older students before the NDP for the first few years at school. It is possible that using a control group that was more closely matched age-wise to the intervention group might have revealed in greater differences. However, Figure 1 also shows that, by year 5, there was an obvious advantage in participating in the NDP.

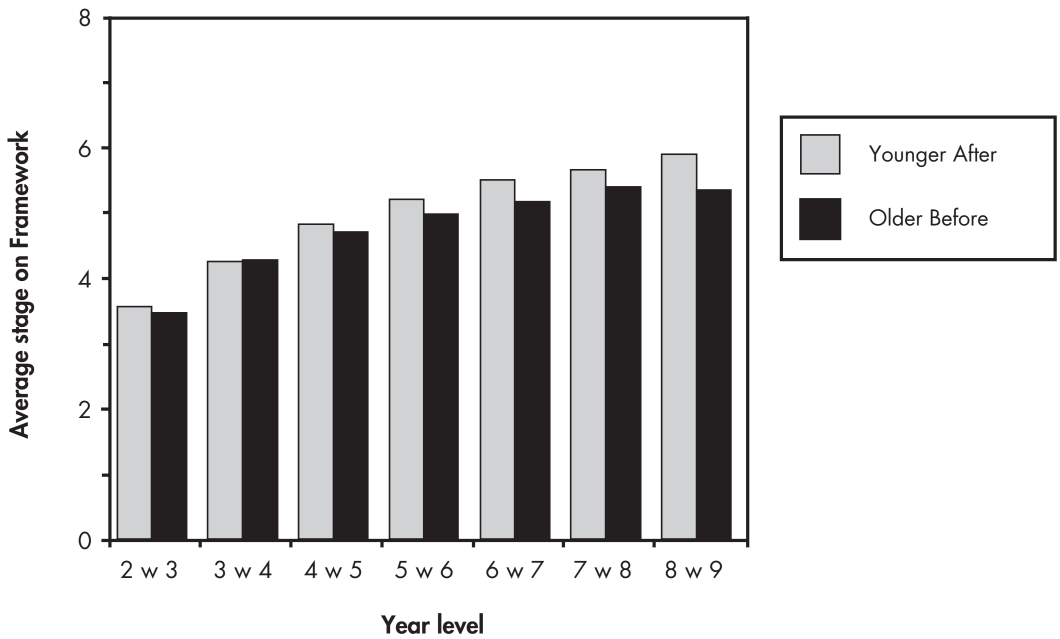

Figure 1: Average stage on the **additive** domain for comparisons between younger year group after a year of the NDP and older year group before they began the NDP. Note: the "w" in the horizonal axis labels for this and following figures means "with".

Figures 2 and 3 show the comparisons for the multiplicative and proportional domains between younger students after participating in the NDP and older students before participating in the NDP.

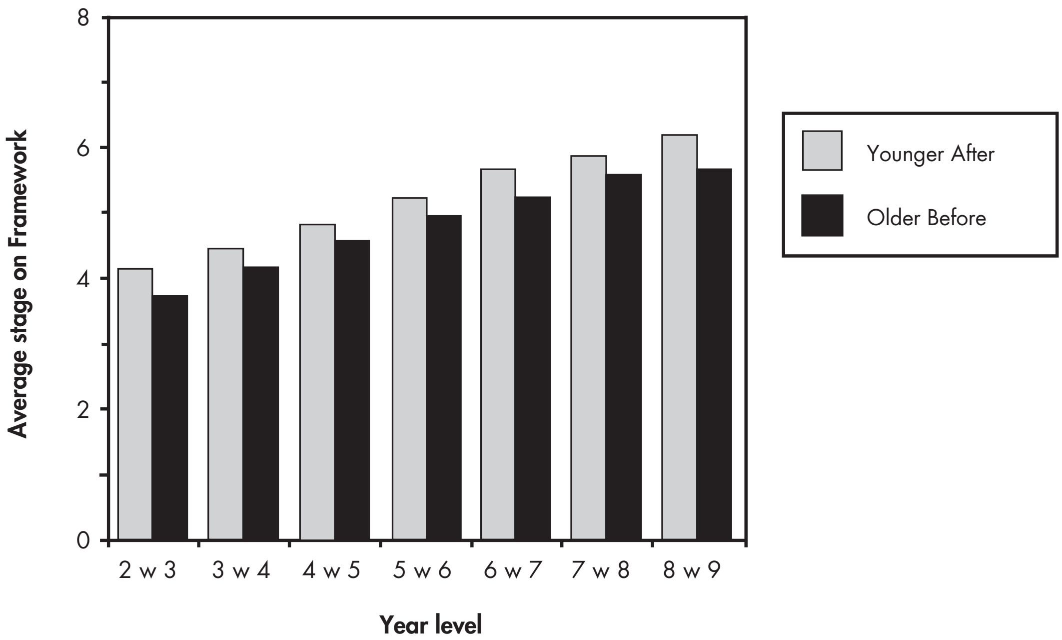

Figure 2: Average stage on the **multiplicative** domain for comparisons between younger year group after a year of the NDP and older year group before they began the NDP

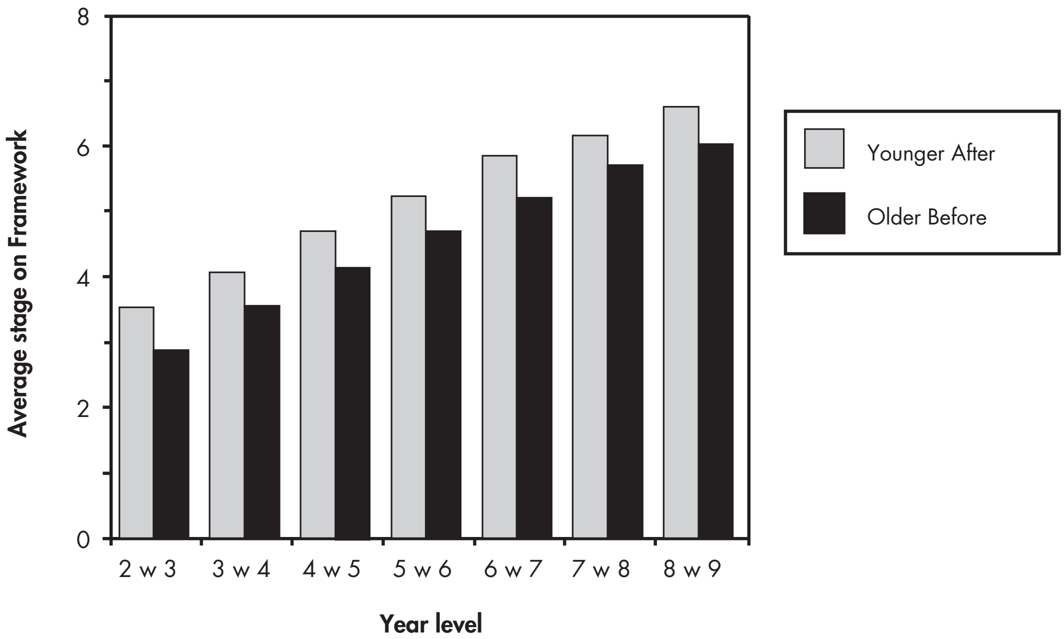

Figure 3: Average stage on the **proportional** domain for comparisons between younger year group after a year of the NDP and older year group before they began the NDP

It is clear from Figures 2 and 3 that there is an advantage in participating in the NDP right from year 2, but this may have been the result of comparing a small number of mathematically proficient students at that year level with a larger more heterogeneous group of slightly older students (see Appendix B (p. 155), which shows that the proportion of students for whom data was not entered or not applicable decreased with year level for all but Numeral Identification, where the opposite pattern was found). The effect was particularly pronounced for the proportional domain.

The Differential Impact of the NDP on Students from Different Subgroups

Appendix D (p. 160) shows the data used to calculate effect sizes for each of the comparisons for the cohort overall and for different subgroups between younger students after participating in the NDP and older students before participating in the NDP. The magnitude of these effect sizes indicates how much students who participate in the NDP benefit relative to slightly older students from the same subgroup who had not yet begun the NDP. It is clear from Appendix D that some of the effect sizes are quite large (that is, half a standard deviation or greater) and that certain subgroups had more of these larger effect sizes than others. For example, 11 out of the 42 effect sizes for Pasifika students were half a standard deviation or greater (see Figure 4, which shows Pasifika students on the multiplicative domain), and this was also the case for students from low-decile schools. In contrast, only five out of 42 effect sizes for European students were half a standard deviation or greater, and the same was true for Māori students and students from high-decile schools. Effect sizes of 0.40 or greater were most frequent for the adjacent year-group comparisons of Pasifika students (25 of 42) and students from low-decile schools (20 out of 42). Effect sizes of 0.40 or greater were least frequent for European (12 of 42), students from high-decile schools (14 of 42), and Māori students (15 of 42). Effect sizes were greater for the multiplicative and proportional domains, with 63% (multiplicative) and just over 77% (proportional) of effect sizes greater than one-third of a standard deviation. Some of the effect sizes for Māori, Pasifika, and low-decile students were more than three-quarters of a standard deviation (the highest effect size for each group was 0.87, 0.82, 0.76, and 0.61 for low-decile, Māori, Pasifika, and European students respectively).

A summary of Appendix D is shown below in Table 2, with the effect sizes averaged across 2005 and 2006 and across the comparisons for adjacent years (from year 2 with year 3 to year 8 with year 9) for the additive, multiplicative, and proportional domains. The overall average across the three domains was 0.34. The average effect sizes were above the overall average for Pasifika students (0.40), Māori (0.35), and students from low-decile schools (0.38). This data suggests that the achievement gap, while not necessarily narrowing as a result of participation in the NDP, was prevented from becoming larger, and this effect was greatest for Pasifika and students from low-decile schools, on average.

Table 2
Summary of Effect Sizes for Younger Students after the NDP and Older Students before the NDP Averaged across Comparisons between Adjacent Years from Year 2 to Year 9 and across 2005 and 2006

Comparison	Additive	Multiplicative	Proportional	Average Overall
Years 2–9				
Overall	0.24	0.39	0.39	**0.34**
European	0.24	0.38	0.37	**0.33**
Māori	0.24	0.40	0.42	**0.35**
Pasifika	0.26	0.45	0.48	**0.40**
Low Decile	0.24	0.44	0.45	**0.38**
High Decile	0.27	0.38	0.38	**0.34**

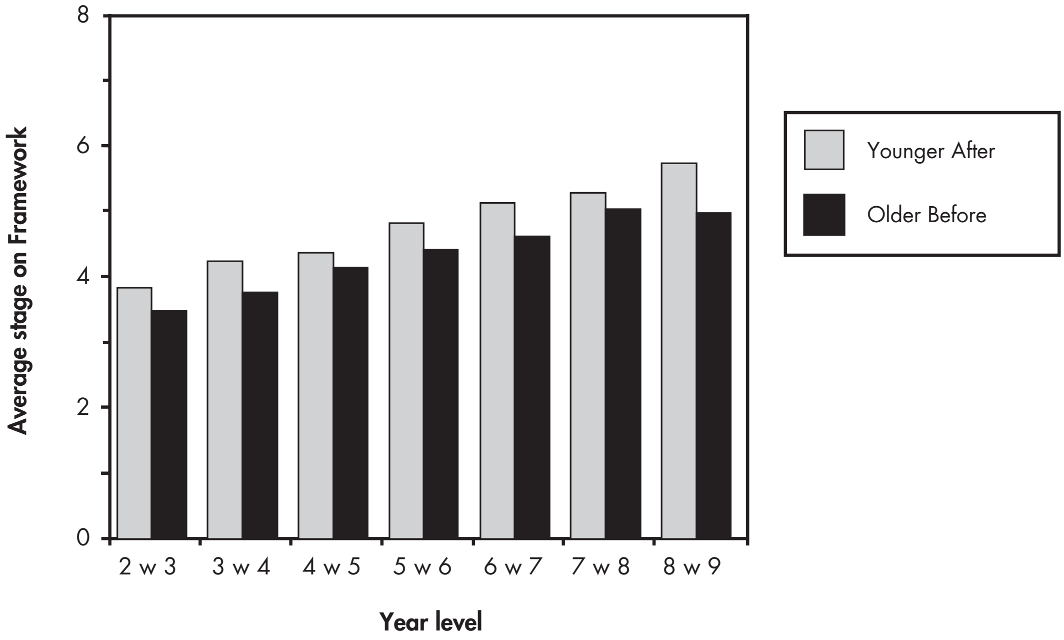

Figure 4: Average stage on the **multiplicative** domain for comparisons between **Pasifika** younger year group after a year of the NDP and older year group before they began the NDP

Patterns of Progress

Appendix E (p. 165) shows the percentages of students who progressed to higher stages relative to their initial stage on the Framework as a function of ethnicity. This analysis compares the progress of students from each subgroup, all of which began at an identical stage on the Framework. In general, European students made greater progress than Māori or Pasifika students. However, for some comparisons, Pasifika or Māori made the greatest progress. For example, for those students who began the NDP at stage 3 or below on the additive domain, Pasifika students made the greatest progress, with 72.0% moving up to a higher stage compared with 71.2% of European and 69.3% of Māori students (see Figure 5). On the multiplicative domain, Pasifika students made greater progress than Māori students when their initial stage on the Framework was 4, 5, or 6 (see Figure 6).

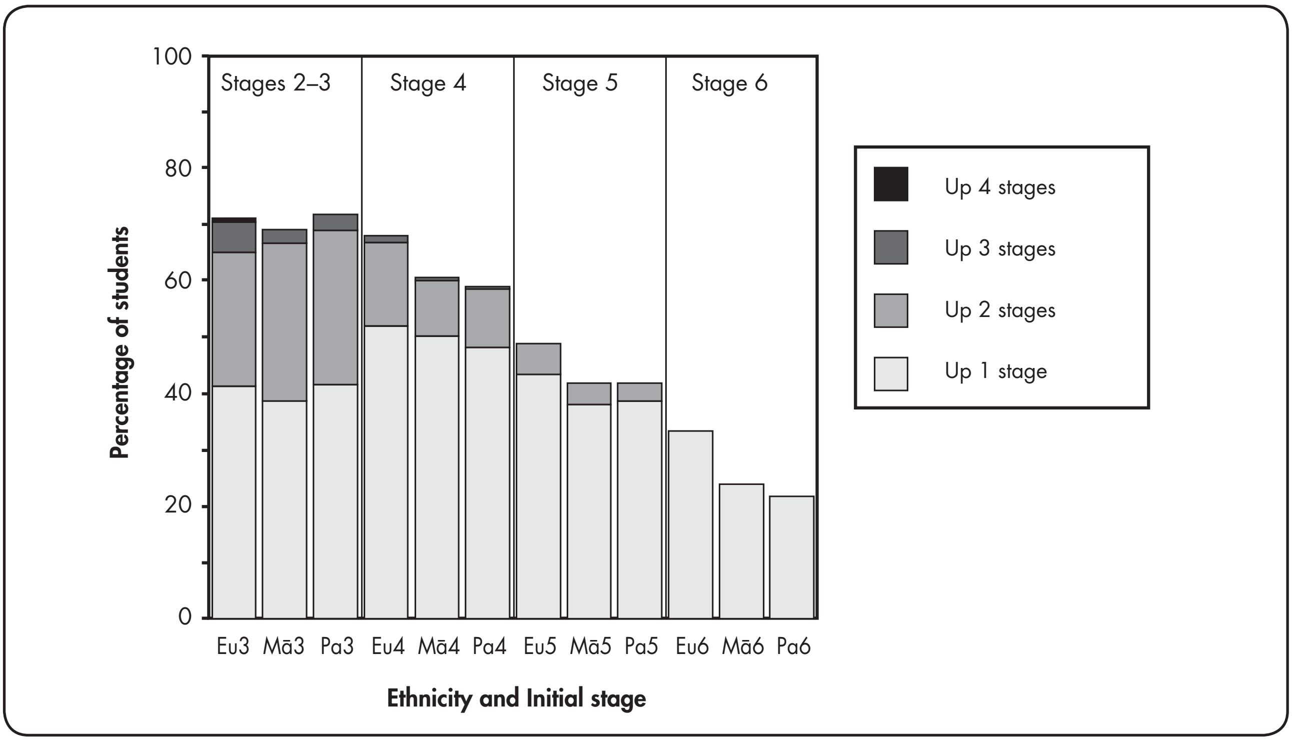

Figure 5: The percentage as a function of **ethnicity** of year 5–9 students who progressed to a higher stage on the **additive** domain relative to their initial stage on the Framework

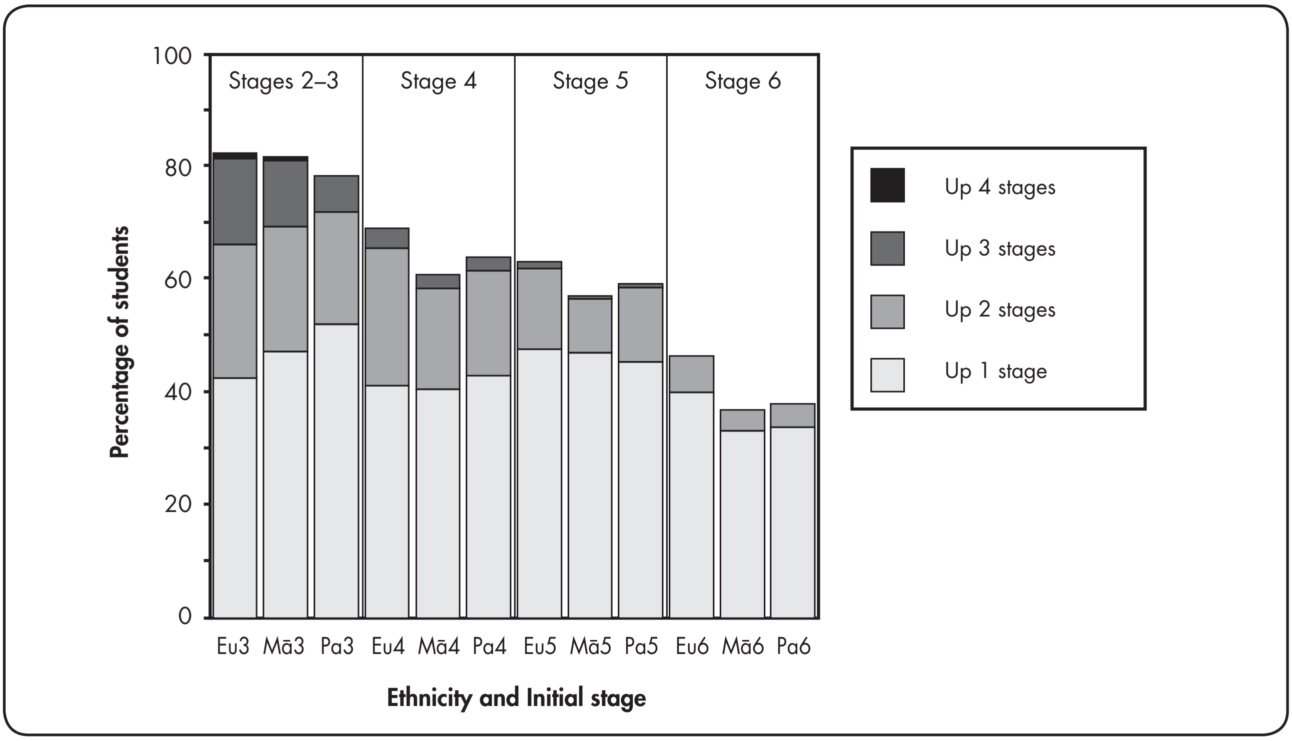

Figure 6: The percentage as a function of **ethnicity** of year 5–9 students who progressed to a higher stage on the **multiplicative** domain relative to their initial stage on the Framework

Appendix F (p. 168) shows the percentages of students who progressed to a higher stage relative to their initial stage on the Framework as a function of school-decile level. The patterns are fairly consistent, with students from high-decile schools making the greatest progress and those from low-decile schools making the least, relative to the same initial stage on the Framework.

Looking at Patterns over Time

Figure 7 shows the percentage of years 5–8 students at stage 7 or higher on the multiplicative domain at the beginning and end of a year on the NDP between 2002 and 2006. It is clear from Figure 7 that only a very small proportion of year 5 students reached stage 7 or higher on this domain. However, the improvements appear to increase with year level, as can be seen in the magnitude of the difference in proportion of students at stage 7 or higher finally compared to initially. Year 6 and year 7 were very similar, but there was a substantial improvement in year 8.

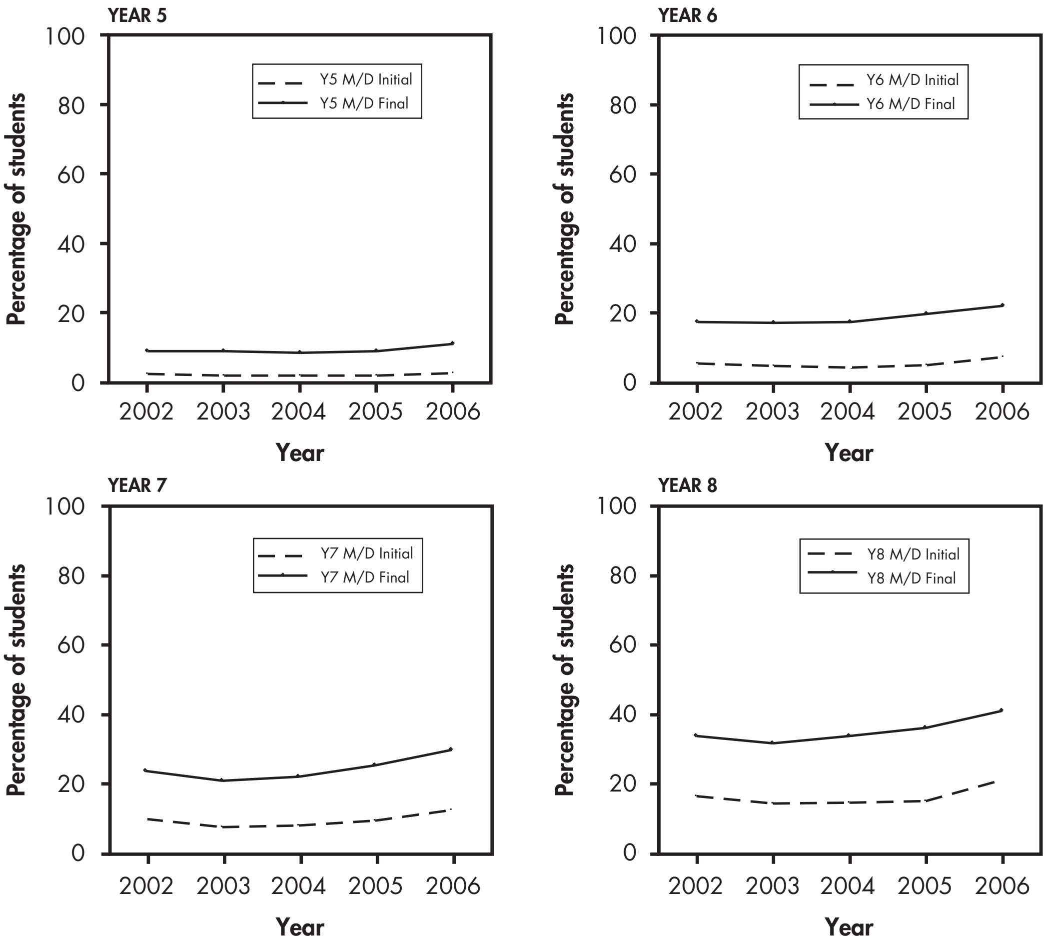

Figure 7: The percentages of year 5–8 students at stage 7 or higher on the **multiplicative** domain at the beginning (initial) and end (final) of a year of the NDP between 2002 and 2006

A similar pattern is evident in Figure 8, which shows the proportion of year 5–8 students at stage 7 or higher for the multiplicative and proportional domains by the end of a year on the NDP. It is clear from Figure 8 that the gaps between years 5 and 6 and between years 7 and 8 were greater than the gap between years 6 and 7. The reasons for this pattern are not obvious. It could be that a change of school for many students at the beginning of year 7 requires a major adjustment that slows down their progress in mathematics learning. By the end of the following year, students have had a chance to settle in to their new environment and adapted to the different expectations at intermediate school (for example, less emphasis on method), and as a result, their progress returns to former levels. This warrants further investigation.

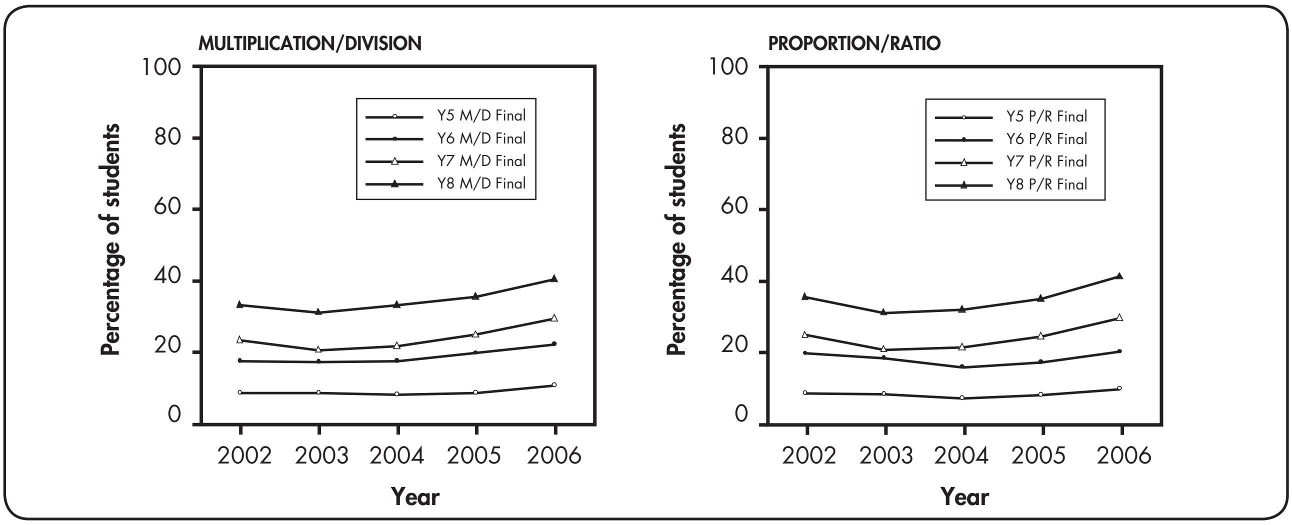

Figure 8: The percentages of year 5–8 students at stage 7 or higher on **multiplicative** and **proportional** domains at the end of a year of NDP between 2002 and 2006

The Challenges of Becoming a Multiplicative Thinker

Appendix G (p. 171) shows comparisons between students who started below stage 7 but reached stage 7 or higher by the end of a year of the NDP and those who were initially below stage 7 but did not progress to stage 7; these are shown for each of the three domains of the Framework. The biggest difference between those who reached stage 7 on the additive domain and those who did not was in their performance at stage 8 on the multiplicative domain, which almost one-third (30.4%) of the students who made progress reached, compared with only 5.4% of the no-progress students. As the stage 8 multiplicative domain tasks involved doing division with decimals and the stage 7 additive domain tasks involved doing subtraction with fractions and decimals, this result is not altogether surprising.

A similar pattern was found for the differences between those who reached stage 7 on the proportional domain and those who did not. On the multiplicative domain, what distinguished those who reached stage 7 from those who did not was their performance on the proportional domain, where more than two-thirds (68.2%) of the students who made progress reached stage 7 or higher, compared with only one-quarter (28.1%) of no-progress students.

Focus on Students Experiencing Difficulties: Persistent Counters

Appendix H (p. 173) presents the percentages of year 5–9 students who continued to use counting strategies for additive domain problems and compares these results with the corresponding percentages of students who reached stage 5, early additive part–whole thinking, by the end of a year on the NDP. The fact that 3846 students persisted in using counting, despite the best efforts of their teachers to help them acquire part–whole strategies, is of considerable concern. Comparison of these students with those in the same year group who had reached stage 5 shows that important components of knowledge are missing or are very weak in persistent counters. For example, knowledge of place value, basic facts, and number sequence forwards and backwards were areas where students at stage 5 showed considerably greater competence than did persistent counters (those at stages 0 to 4). Close to two-thirds of year 7 students (62.0%) were below stage 5 on place value knowledge, whereas only one-quarter (25.6%) of year 7 students were able to use simple part–whole strategies to solve addition and subtraction problems. More than half (55.6%) of the year 5 persistent counters were at stage 4 or lower on basic facts, whereas only one-fifth (21.3%) of year 5 students were able to use simple part–whole strategies to solve addition and subtraction problems.

The Relationships between Basic Facts Knowledge and Proficiency on Other Domains

Appendix I (p. 174) shows the percentages of year 5–9 students at various stages on the basic facts domain of the Framework after a year of the NDP and the stages they had reached on other domains of the Framework in that time. More than 3000 students were still at stage 4 or below on basic facts, despite a year of the NDP. A higher proportion of these students were in year 5 (37.3%), between one-fifth and one-quarter of the students were in years 6 and 7 (20.2% and 23.7% respectively), and only small proportions were in years 8 and 9 (13.5% and 5.2% respectively). These students knew some small number combinations for sums totalling 10 or less (for example, 2 + 3, 5 + 4, 6 + □ = 10), some single-digit doubles (for example, 6 + 6, 9 + 9), and sums of 10 combined with single-digit quantities (for example, 10 + 4, 7 + 10). However, what distinguished these students from those at stage 5 was that they were unable to combine different addends to get totals between 10 and 20 (for example, 8 + 6, 6 + 9) or to recall number facts from the five-times table (for example, 8 x 5, 5 x 7). Nor were they able to subtract single-digit quantities from "teen" numbers (for example, 17 – 9, 15 – 6) or to recall other multiplications (6 x 7, 8 x 4), all of which are characteristics of students at stage 6. Recall of division facts (for example, 56 ÷ 7, 63 ÷ 9), a feature of stage 7 students, was also beyond their capabilities.

An examination of performance on the operational domains of students below stage 5 on basic facts showed that the majority of these students solved addition/subtraction problems by counting on (40.3% were at stage 4) or by simple partitioning and recombining of quantities (45.0% were at stage 5). Only 7.2% reached stage 6 (advanced additive part–whole) or higher. On the multiplicative domain, these stage 2–4 students tended to either skip count (44.6% were at stage 4) or used repeated addition (27.2% were at stage 5). On the proportional domain, half of them were able to share 12 beans into thirds (50.8% were at stages 2–4) or work out that if 4 + 4 + 4 = 12, then one-third of 12 beans is 4 beans (27.8% were at stage 5). About one-third of them could name unit fractions (33.9% were at stages 2–3), and another third could order unit fractions (38.3% were at stage 4). Between half and two-thirds of them could count by tens to work out how many $10 notes would be needed to buy items costing $80 and $230 (60.9% were at stage 4). It was interesting to note that 9.8% had reached a higher stage on basic facts at the initial assessment. However, this might have been the result of their teachers' inexperience with the assessment tool at the beginning of the PD programme. Whereas only 40% of them had reached stage 4 on basic facts initially, after a year of the NDP this had increased to 75.2%.

Students who were assessed as being at stage 5 on the basic facts domain were more likely than those at stages 2–4 to be able to reach stage 6 on other domains, but only about one-third (34.7%) of them reached stage 6 on the multiplicative domain, and about one-quarter reached stage 6 on the additive and proportional domains (25.1% and 24.6% respectively). Only about one-sixth reached stage 6 on the fractions or place value domains (17.7% and 16.9% respectively). Approximately half (between 45.0% and 60.9%) of the students at stage 6 on the basic facts domains reached stage 6 or higher on the other domains. Of those who reached stage 7 on basic facts, between 75% and 90% reached stage 6 or higher on other domains. Students needed to be at stage 8 on basic facts (able to recall division facts as well as addition, subtraction, and multiplication facts) to be virtually guaranteed of reaching stage 6 or higher on other domains on the Framework.

The Relationships between Place Value Knowledge and Proficiency on Other Domains

Appendix J (p. 176) shows the percentages of year 5–9 students at various stages on the place value domain of the Framework after a year of the NDP and the stages they had reached on other domains of the Framework in that time. It is clear from Appendix J that until students are able to count by tens (stage 4), they have little chance of succeeding on any but the simplest addition/subtraction problems. Those at stage 5 on the place value domain (who were able to give the number of tens in

230 and could identify 6.8 on a number line) were more likely to be able to use at least two different mental strategies to solve multi-digit problems from the additive or multiplicative domains or to find a fraction of a number (between 42% and 55% of the students, depending on the domain). Substantially more of the students at stage 6 (who were able to give the number of hundreds in 26 700 and find the number three-tenths more than 4.8) could do this (between 69% and 82% of the students). Close to 90% or more of students at stage 7 on the place value domain (who were able to find the number of tenths in 4.67 and order decimals of varying lengths) were at stage 6 or higher on the other domains. Students at the highest stage on the place value domain (stage 8) were able to find the number of hundredths in 2.097, round 7.649 to the nearest tenth, give three numbers between 7.59 and 7.6, and name 137.5% as a decimal. These students had such an extensive understanding of the number system that they tended to be at stage 7 or higher on all other domains. Students needed to be at stage 8 on the place value domain (able to convert between fractions, decimals, or percentages) to be virtually guaranteed of reaching stage 6 or higher on other domains on the Framework. This is consistent with Ross's (1989) assertion that to understand place value, students must co-ordinate and integrate knowledge about the notational system used to record numbers as well as about part–whole relationships among numerical quantities. Coming to understand these ideas is difficult, and the concepts develop slowly over a number of years (Ross, 1989; Verschaffel, Greer, & De Corte, 2007). As Appendix B (p. 155) shows, even by the end of year 9, only about one-third of students have a well-developed understanding of decimals (that is, the students were at stages 7–8).

General Discussion

The findings presented here include some good and some not-so-good news. It is heartening to see that Pasifika students particularly (and Māori students also) did better as a result of the NDP than they would have otherwise. This is evident in the larger effect sizes found when comparing both younger Pasifika students after a year of the NDP and slightly older Pasifika students before they began the NDP with corresponding European students. The reasons for the larger effect sizes for Pasifika are not entirely clear. It may be that a combination of the schooling improvement initiatives that have been in place now for several years (for example, the Manurewa Enhancement Initiative, see Young-Loveridge, 2005) and recent home–school partnership projects with Pasifika communities have helped schools serving low-decile communities improve the mathematics learning of their students. The schooling improvement initiatives have provided teachers with support above and beyond the NDP itself. As Sowder (2007) has pointed out, "the key to increasing students' mathematical knowledge and to closing the achievement gap is to put knowledgeable teachers in every classroom" (p. 157).

The percentages of students overall reaching the upper stages on the Framework on the multiplicative and proportional domains after a year of the NDP were considerably lower than those reflected in the achievement objectives of the draft New Zealand Curriculum (see Ministry of Education, 2006). For example, according to level 3 of the draft curriculum, most year 6 and 7 students should have a flexible range of additive strategies for dealing with addition and subtraction problems, yet fewer than half of the year 6 students and barely half of the year 7 students in the 2006 cohort reached stage 6, advanced additive part–whole thinking, by the end of the year. At level 4 of the draft curriculum, most year 8 and 9 students are expected to have a flexible range of multiplicative strategies for dealing with multiplication, division, and fraction problems. Yet only between one-third and one-half of year 8 and year 9 students reached stage 7, advanced multiplicative, by the end of the year. The expectation that students at level four should be multiplicative thinkers is based on research evidence showing that students cannot engage with algebra effectively if they are not multiplicative thinkers (for example, Lamon, 2007; Wu, 2002). Hence, it is important not to compromise the expectations for particular curriculum levels.

The mismatch between the achievement objectives in the draft curriculum and the proportion of students at particular Framework stages found in this analysis signals the need for further intensive efforts to improve mathematics teaching and learning at the upper primary and intermediate levels.

The Ministry of Education's fee subsidy scheme, which provides some financial support to offset the costs of teachers doing further university study in mathematics education, may help, but far more publicity is needed as well as support from schools if the scheme is to have an appreciable impact on teachers' understanding of the upper stages of the Framework. A revised version of *Book 1: The Number Framework* was published earlier this year (see Ministry of Education, 2007a), and the combination of array and number-line models more effectively captures the complexity of multiplicative thinking in particular. More recently, a revised version of *Book 6: Teaching Multiplication and Division* was released (see Ministry of Education, 2007b). It remains to be seen whether these revised books make a difference to teachers' understanding of the upper stages on the Framework. As Young-Loveridge, Taylor, Hāwera, & Sharma (this volume) point out, fractional number and multiplicative thinking are extraordinarily complex, and it might be naïve to expect teachers to acquire a deep and connected understanding of these areas after only one or two years of professional development. As Ward, Thomas, and Tagg (this volume) have shown, even teachers who have worked with the NDP over several years do not necessarily have a strong understanding of fractional number. Consistent with this are the findings of Lamon's (2007) research showing that it took considerably more than two years for new ways of teaching fractional number to have a beneficial impact on students' understanding, and that was when the focus was explicitly on fractional number.

It is not just a matter of improving teachers' subject matter knowledge of mathematics or their pedagogical content knowledge in mathematics (Ball, Hill, & Bass, 2005; Ball, Lubienski, & Mewborn, 2001; Moch, 2004; Shulman, 1986). Teacher's beliefs about mathematics learning, and in particular whether they have a *conceptual orientation* or a *calculational orientation*, may have a substantial impact on the way that they teach mathematics in their own classrooms (see Philipp, 2007). A conceptual orientation leads to different kinds of classroom discussions from a calculational orientation. A teacher with a calculational orientation is interested in the calculations students perform to get their answers, whereas a teacher with a conceptual orientation is more interested in students' explanations of their reasoning (Philipp, 2007). According to Hiebert and Grouws (2007), conceptual understanding is promoted by teaching that draws students' attention explicitly to concepts (including the connections between mathematical facts, procedures, and ideas) and ensures that students "struggle" with important mathematical ideas (that is, they expend effort in making sense of mathematics and working out something that is not immediately apparent). It would be interesting to know the extent to which teachers participating in the NDP move to a conceptual orientation.

The NDP's increased emphasis on communicating mathematical ideas and reasoning is not without problems. As previous research has shown, students whose teachers have participated in the NDP don't necessarily value the opportunities to share their strategies with other students or learn from others' approaches (see Young-Loveridge, Taylor, & Hāwera, 2005). Several researchers (for example, Lubienski, 2007; Zevenbergen, 2001) have found that students from lower socio-economic backgrounds are not as comfortable as students from higher socio-economic backgrounds in contributing to discussions about their ways of reasoning about mathematics. However, Hunter (2005, 2006) has shown that teachers can help students in low-decile schools learn how to contribute to a community of inquiry and can assist them to become effective communicators about their mathematics thinking and learning. More work may need to be done in this area to help narrow the gap between European and Māori/Pasifika students and between students at high- and low-decile schools.

The analysis presented here suggests that knowledge of basic facts may be an important requirement for using an advanced part–whole strategy (stage 6) on the operational domains. These findings are consistent with those of other researchers who have investigated the relationship between arithmetical fact retrieval and general measures of arithmetic performance (Cumming & Elkins, 1999; Dowker, 2005; Geary & Brown, 1991; Geary, Brown, & Samaranayake, 1991; Ostad, 1998). Memory processes and the inhibition of incorrect responses seem to play an important part in number fact retrieval (Barrouillet, Fayol, & Lathuliere, 1997). However, researchers have shown that it is possible to improve students' memory for number facts with training (for example, Dowker, 2005; Pauli, Bourne, & Birbaumer, 1998). The importance of basic fact knowledge was highlighted in the latest NEMP (National Education Monitoring Project) study (Flockton, Crooks, Smith, & Smith, 2006), which showed a moderate overall decline (a 5% difference) in average performance on number task components at the year 4 level for the 2005 cohort, compared with that of 2001 (see Crooks & Flockton, 2002). The NEMP researchers have attributed the overall decline to the 9% decline on the large number of task components (71) that involved recall of facts or simple calculations with the four basic operations. This conclusion was supported by their finding of a drop (from 56% to 36%) in the proportion of year 4 students who said they practised mathematics facts or tables in their own time, which the NEMP researchers took as a clear indication of a reduced emphasis on basic facts by teachers.

However, it is important to acknowledge the possibility that students tried to work out the answers using some kind of strategy (either a counting-on or a part–whole strategy), which meant that they often missed the presentation of the next item (4 seconds later) because they weren't quite ready for it. The tasks were presented by the computer and not monitored by an adult, so it is impossible to know just how many of the NEMP problems were solved by the straight recall of basic facts and how many were solved by counting on or by deriving the answer using knowledge of a different basic fact. This warrants further investigation through a probe study. Gray and Tall (1994) use the term "procept" to refer to the amalgamation of both process and concept and the way that multiple procepts can represent the same object (for example, "6" as 1, 2, 3, 4, 5, 6, or ..., 4, 5, 6, or 3 + 3, or 4 + 2, or 10 – 4). They distinguish between meaningful known facts that are generated by flexible thinking and facts are are remembered simply by rote. It is the flexible thinking that is reflected in derived facts that should be the goal of mathematics teaching, rather than the simple recall of facts without meaning. Interestingly, the 2005 year 4 NEMP students outperformed the 2001 cohort (by 3%) on more complex problem-solving tasks, including algebra, logic, finding patterns, estimation, and identifying sequences, despite the lower socio-economic status of the cohort (compared to the 2001 sample, the 2005 NEMP sample included considerably fewer year 4 students from high-decile schools [32% vs 41%], and slightly more year 4 students from low-decile schools [28% vs. 27%]).

Even though most primary schools have now been given an opportunity to participate in one of the PD programmes of the NDP, there is an ongoing challenge to continue teachers' professional learning about mathematics teaching and learning. An enormous amount of effort has been put in to developing the people who can work with teachers to bring about change in their ways of approaching the teaching and learning of mathematics. It is hoped that this effort can continue, so that the gains that have been made are not lost in the future. It is clear from the results that teachers need a great deal more help in coming to understand multiplicative thinking and proportional reasoning.

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The Development of Algebraic Thinking: Results of a Three-year Study

Kathryn C. Irwin
University of Auckland
<k.irwin@auckland.ac.nz>

Murray S. Britt
University of Auckland
<m.britt@auckland.ac.nz>

Analysis of three years of results from the same students in years 8, 9, and 10 shows a steady increase in algebraic thinking. One pair of schools outperformed the other three pairs on each occasion. The intermediate students from this pair of schools entered secondary school with a better understanding of algebraic thinking than did students from the other schools. The use of this thinking was encouraged by teachers in the secondary school despite that school not being in the Secondary Numeracy Project. An additional analysis showed that students who had reached stage 6 or higher in the Numeracy Development Project showed the greatest ability at generalising algebraic thinking. That is, they were most able to express concepts with letters on the basis of knowing how to operate flexibly with numbers.

Background

In 2004, we embarked on a three-year study of specific students as they moved from intermediate schools that had been involved in the Intermediate Numeracy Project (INP) to secondary schools that might or might not be using the Secondary Numeracy Project (SNP) (see Irwin & Britt, 2005, 2006). Our hypothesis was that students with a good understanding of the legitimate manipulation of numbers for addition, subtraction, multiplication, and division would be able to generalise this knowledge to algebra that used letters to state generalities. In the words of Fujii and Stephens (2001), these students had been using numbers as quasi-variables and therefore should move without too much difficulty to the use of letters as variables.

Our findings showed that some students in intermediate school could make this generalisation even before they were introduced to algebraic symbols. Overall, when assessed on the same test, students improved in the first two years of secondary school in their ability to make this generalisation. However, there were marked differences among students and among schools.

Results obtained in this third year confirm this trend in improvement. A summary of findings for all three years of this study is given at the end of the paper.

Method

Assessment Instrument

We chose to assess algebraic thinking in the context of compensation. The same test was given in all three years. It asked students to use, for each of the four operations, a method demonstrated by hypothetical students. The method required them to generalise from specific examples for using the compensation for each operation. For example, in addition, the same number is added to one addend and subtracted from the other addend in order to keep the sum the same, while in subtraction, the same number must be added or subtracted from both the minuend and the subtrahend for the compensation to work. In addition to demonstrating this generalisation with numbers, students were then asked to express this relationship with letters.

The first item for each operation involved demonstrating this generalisation with whole numbers. The second item required the students to demonstrate the operation with a decimal fraction. The inclusion of items that included a decimal fraction came out of a previous study (Irwin & Britt, 2004). The third item asked them to show how this generalisation would work when the number to which compensation was applied was a letter, the fourth item asked them the same question when the operations involved letters and a decimal fraction, and the final item asked them to represent the second part of an equation that used only letters.

The test given is shown in full in Appendix K (p. 178). Others are welcome to use this test, but we request that it either be noted as the Britt Algebraic Thinking Test or reference made to this paper in anything that is written about the test.

In addition to using this Algebraic Thinking Test, one analysis used results of strategy-scale scores of the Numeracy Project Assessment (NumPA) that teachers gave to all participating students. The assessment results used were from 2005 when the students were in year 9 (available on www.nzmaths.co.nz/numeracy/SNP/Assessment/FullSNumPA.doc).

Participants

Students from four pairs of contributing intermediate and secondary schools agreed to participate in this study. Two pairs of schools were in the Wellington area and two pairs were in the Auckland area. A general description of the secondary schools is given in Table 1. (Note that all students in the third year of the study were now at secondary school.)

Table 1
Characteristics of Secondary Schools that Participated in the Assessment of Algebraic Thinking in 2006

School number	Secondary School decile*	Number of students*	Number of students for whom 3 results were available	In Secondary Numeracy Project (SNP)
1	4	726	13	Yes
2	4	1 201	14	Yes
3	6	1 583	61	No
4	8	1 314	28	Yes
Total		4 824	116	

*Decile and number of students as given on TKI website

Procedure

Boxes of tests were couriered to each of the participating schools in 2006, as in previous years. Each Head of Department (HOD) Mathematics was asked to have teachers give the test to all year 9 and 10 classes in their school during term 4 of the year. Teachers were to read to students the instructions that appear on the first page of the Appendix (p. 178). Tests were then returned to the authors for marking. The tests were marked, under supervision, by students doing the graduate diploma in secondary mathematics education. The main instruction was that an item be marked correct only if the compensation method was correct for that operation; items with the correct answer but showing no evidence of generalisation of the method of compensation were to be disregarded.

In some cases, schools chose not to give the test to all classes. For example, in 2006, secondary school 1 chose not to give the test to their three lowest year 9 classes and secondary school 4 provided results for 114 fewer year 10 students in 2006 than in 2005. The number of students assessed differed markedly between 2004 and 2006 in three of the four schools. The only effect of these differences was to reduce the number of students in the longitudinal data set.

Results

Three analyses were carried out to evaluate the data gathered from the tests of algebraic thinking. The first analysis explored the correlation between individual students' scores for the three strategy scores of the NumPA at the end of year 9 and scores on the Algebraic Thinking Test. The second analysis looked at the Algebraic Thinking Test scores for all year 9 and year 10 students in 2004, 2005, and 2006. The third analysis was a comparison of the means of students in each school who took the Algebraic Thinking Test in all three years.

Analysis 1

In 2005, the scores that year 9 students gained on the Algebraic Thinking Test and on NumPA were correlated for the three secondary schools that were in the SNP. For this analysis, all students in year 9 for whom both scores were available were included. In the diagnostic use of scores on strategy scales, additive, multiplicative, and proportional thinking are reported separately. However, we reasoned that all three skills were necessary for the development of algebraic thinking and therefore added the three scores together. This procedure also provides a wider range of scores and is therefore useful for correlations. Correlations are given in Table 2.

Table 2
Correlation of the Total of Three Strategy-scale Scores on the NumPA and the Algebraic Thinking Test at the End of Year 9 in 2005

School	N	Correlation
1	134	0.66
2	206	0.50
3	–	No NumPA scores
4	217	0.43

All of these correlations demonstrate a significant relationship (<0.01) between NumPA strategies and results on the Algebraic Thinking Test. We can only speculate on the reasons for the difference in correlations in different schools. Some teachers reported that motivation was a problem for some students, and some papers were returned to us with drawings on them. Other teachers reported that they had walked among the students encouraging them to try another page. The NumPA would have been administered individually, a situation in which motivation is usually high. One school had a high portion of Asian and Pasifika students, including ESOL and fee-paying students, for whom understanding or reading English may have been difficult.

A further analysis of the total of the three NumPA strategy scales shows that students who were above the median, 19, on that test were the ones who could demonstrate algebraic thinking with letters. This means that students who were operating at or above the advanced additive stage on all three scales were most likely to transfer this flexible numerical thinking to algebraic thinking.

Analysis 2

All participating schools were given enough test papers to assess all year 9 and year 10 students for all three years of the study, with the exception of secondary school 4, which did not participate in 2004. The number of students assessed in years 9 and 10 fluctuated in all but school 3, as shown in Table 3. Schools appear to have used their own judgment on which classes to assess.

Table 3
Number of Students Assessed in Years 9 and 10 in Each of the Three Years of the Study

Secondary school	Year 9 2004	2005	2006	Year 10 2004	2005	2006
1	180	142	83	0	153	63
2	230	237	224	222	173	233
3	310	338	312	326	322	326
4	–	260	170	–	282	168

The difference in the number of students assessed indicates that it would be wise to compare scores only for school 3. The mean for year 9 and year 10 in Figure 1 shows the mean scores for year 9 and 10 students in secondary school 3 in the three years of the study.

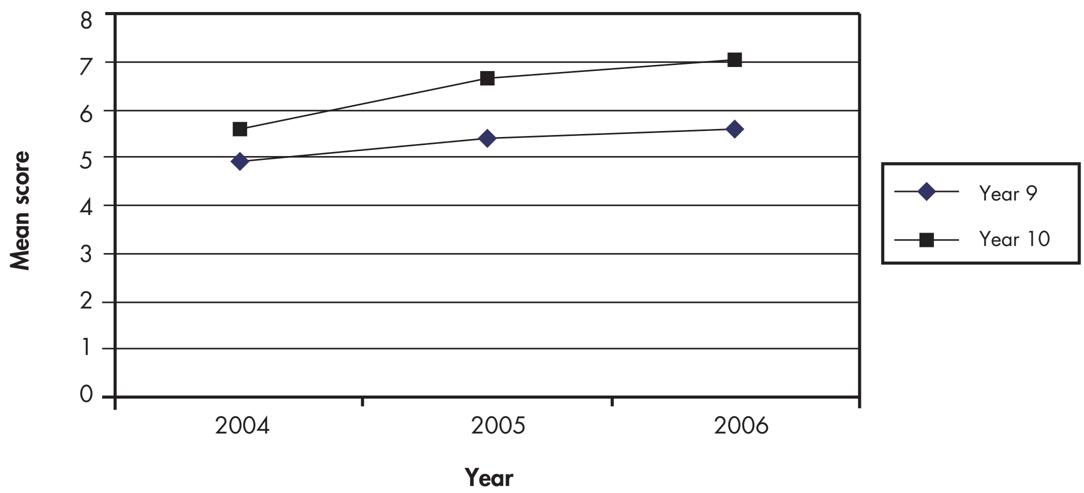

Figure 1: Mean scores for the Algebraic Thinking Test in secondary school 3 over three consecutive years

Note that the mean scores increased for each class in each succeeding year. This may be related to the understanding of the intermediate school cohorts going through or to differences in teaching provided by this secondary school.

Theoretically, it was possible to gain a score of 8 by passing items with numerical quasi-variables (see Appendix, p. 178). To check whether or not students transferred this understanding of quasi-variables to the use of letters as variables, a further analysis was carried out. This analysis looked at the results of the students in school pair 3 to see what percentage of students who passed items in which numbers were used as variables also passed items using letters as variables. The results of this are given in Table 4. The students reported on in this analysis include all the students from intermediate school 3 and all the year 9 and 10 students in secondary school 3 who had attended this intermediate school.

Table 4
Percentage of Students from School Pair 3 Who Passed Some or All Numerical Items and Also Passed Some Literal Items

Year cohort	N	N scoring >0	Percentage passing from 1 to 8 items, some of which included letters	Percentage passing more than 8 items, some of which included letters
Year 8	93	79	6	94
Year 9	208	181	25	75
Year 10	218	203	39	61

Thus it was not necessary for students to pass all of the numerical items for them to be able to generalise the use of numbers as quasi-variables to the use of letters as variables. The ability to generalise to using letters as variables increased with year level.

Analysis 3

This analysis was of the scores for students who took the Algebraic Thinking Test on three occasions. Students who were tested on only one or two of those years were not considered in this analysis.

The students varied both in their initial attainment at the end of year 8 and in their rate of development over the three-year period. An indication of this variability is shown in Figure 2, which presents the results of a random selection of eight students. The figure shows that some students improved over the three years, some did not change, and some declined in performance. In order to take into account this variability among students over each year, a random co-efficient analysis was undertaken. The analysis also accommodated the correlation between responses that arises when the same person is measured on several occasions. These are shown in Table 5.

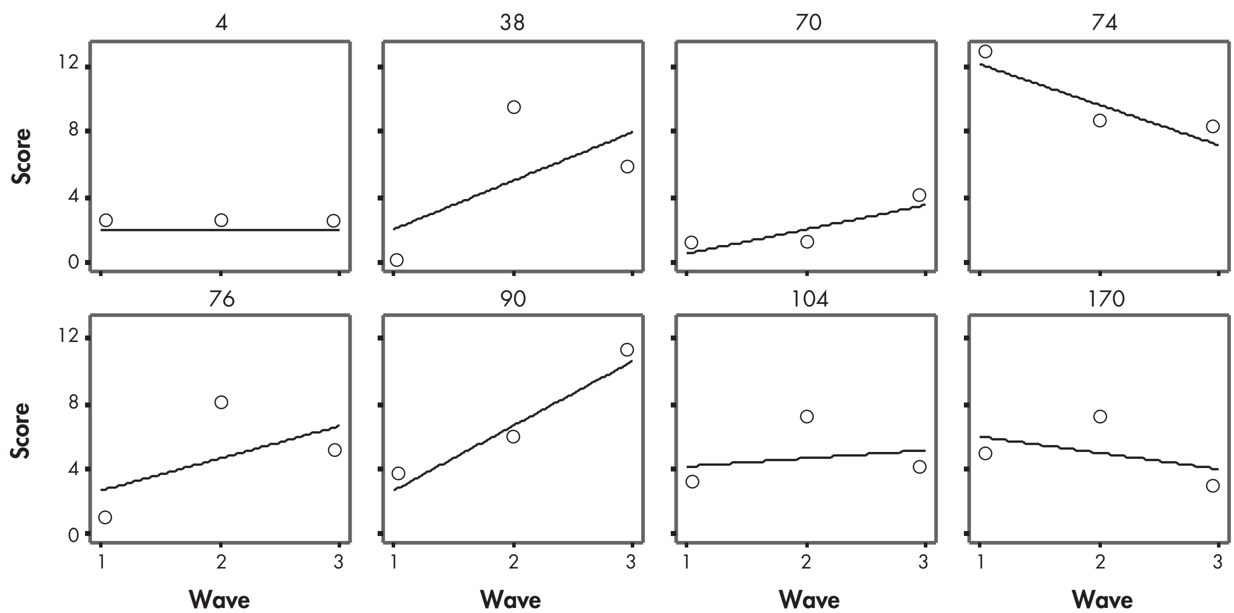

Figure 2: Each panel represents a randomly selected student (ID at top of panel). Waves 1, 2, and 3 refer to years 8, 9, and 10 respectively. The circles show a student's score at each wave, and the straight lines are least-squares fits.

Table 5
Correlation Coefficients among the Scores over Three Years

		Year 8	Year 9	Year 10
Year 8	Pearson correlation	1	0.640**	0.639**
Year 9	Pearson correlation	0.640**	1	0.714**
Year 10	Pearson correlation	0.639**	0.714**	1
	N	116	116	116

**significant at the 0.01 level (2-tailed)

The analysis examined possible differences among students from the four different pairs of schools that they attended.

Figure 3 shows the average performance of all the students who attended a pair of schools over three years. With the exception of school pair 4, the mean scores of students in each of the schools improved over the three years. In examining this figure, it should be borne in mind that the number of students in each pair of schools differed widely: 13 in schools labelled "1", 14 in "2", 61 in "3", and 28 in "4". See Table 1.

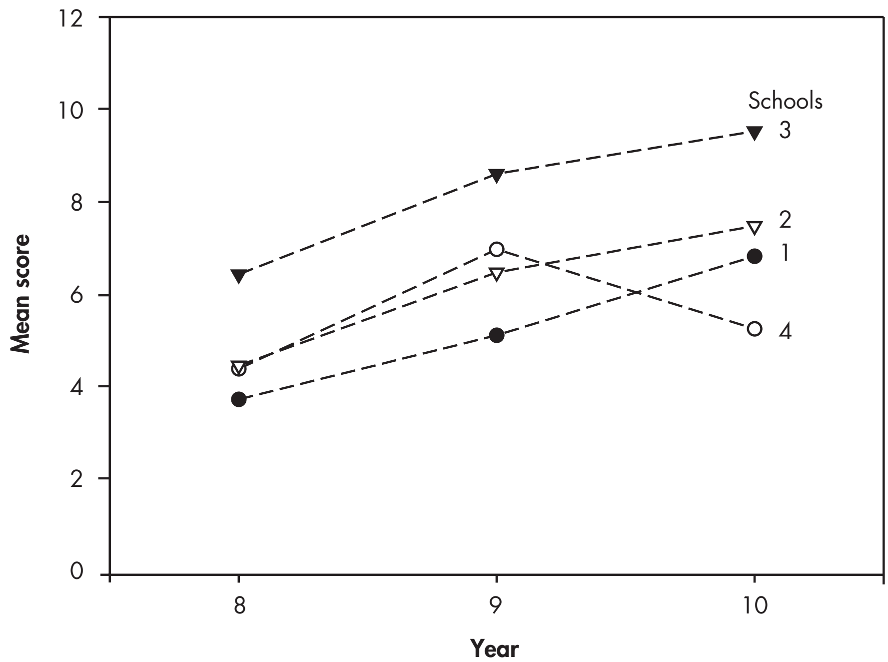

Figure 3: Mean score of students attending each of the four pairs of schools over three years

Scores from school pair 3 were significantly different from school pairs 2 and 4 ($p < 0.05$) but not from school pair 1 ($p = 0.081$) because of the large variance in the small number of student scores from that school pair. The difference among schools when all three years were taken into consideration approached significance $F(3,112) = 2.620$, $p = 0.054$.

A statistical analysis of the data was undertaken by means of SPSS's mixed linear model (SPSS: Statistical Package for the Social Sciences). Two models were fitted. The simpler model estimated, without differentiating among the schools, the average score at year 8 and the average rate of improvement over each subsequent year. The model's estimated score at year 8 was 5.7, and the estimated improvement per year was 1.3 points. This model is shown by the heavy line labelled "all" in Figure 4.

The other model took into consideration the effect of attending different pairs of schools. This model provided a marginally better fit to the data than the simpler one. If this is taken to show a worthwhile improvement in the model's fit, then the resulting estimated initial score and improvement rate for students from each school is shown in Figure 4 by the lines labelled "1", "2", "3", and "4". The effect of attending different schools did not reach significance at the 0.05 level. For both models, the rate of change (1.3 points per year) represented a significant improvement.

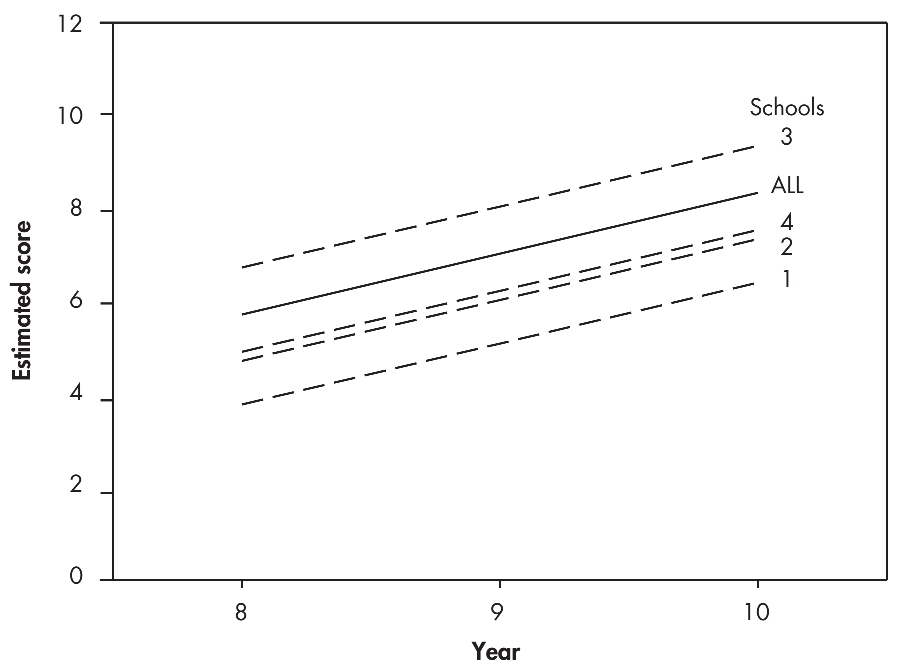

Figure 4: Two best-fitting models: (a) the solid line, labelled "all", shows the estimated score at year 8 and the estimated improvement over three years of the average student, without differentiating among the schools; and (b) a model of the estimated score at year 8 and the estimated improvement of the average student in each of the pairs of schools.

A still more complex model, which allowed for different average rates of improvement for students from different schools, did not improve the fit; it therefore is not discussed further.

Discussion

Three annual reports have now been presented for this study. We discuss first some of the factors that arose out of the analyses in this report. This is followed by a discussion of the one pair of schools, pair 3, which were superior to the other three pairs of schools. In this discussion, we provide information from our interviews with personnel in these schools. Finally, we will draw some conclusions from the data for all three years of this study.

The first analysis presented in this report drew on material not analysed before: the relationship of the NDP and our Algebraic Thinking Test. This test was developed on the assumption that the algebraic thinking we were investigating was developed in the NDP. The first items on each page of this test are similar to exercises suggested in the NDP. For this correlation, we used all the students who had been given both tests at the end of year 9, that is, 557 students. The correlations for each school were high. This demonstrates a definite relationship between the NDP and the Algebraic Thinking Test. Importantly, the students who were judged to be at or above the advanced additive stage, which turned out to be half of the students assessed, were those who were able to transfer their algebraic thinking from the quasi-numerical level, as used in the NDP, to demonstrating this thinking with letters. It is to the credit of the teaching programmes in intermediate and secondary schools that half of students have reached this level by the end of year 9. A further analysis that would be interesting here would be the correlation of strategy scores at the end of year 8 with scores on the Algebraic Thinking Test.

The second analysis looked at all of the results from the four secondary schools for year 9 and year 10 for each of the three years of the study. It appeared that secondary schools assessed most of their students in 2005, the year in which year 9 results were important for the longitudinal study. Only one school, school 3, appeared to have assessed all available year 9 and 10 students in each of three years. When their average results were graphed, both year 9 and year 10 students showed a steady increase across the three years. This is referred to again in the section on this school. The fact that the number of students varied across the years demonstrates one of the potential difficulties of cross-sectional studies that look at different year groups of students and draw comparisons. Failure to assess all students can be one of many reasons why a non-representative sample can turn up in a cross-sectional study.

This difficulty draws attention to the advantage of following the same students across three years. Although 317 students were assessed in year 8, only 116 of these took the Algebraic Thinking Test for three years. The mean for algebraic thinking scores advanced significantly through years 8, 9, and 10 despite differences among the schools. The exception was the 28 students in secondary school 4 whose mean performance was better in year 9 than in the following year. We asked teachers for possible reasons for this decline and were told that one reason might be that teachers had to prepare students for year 11 examinations rather than focusing on algebraic thinking. In looking at the average performance across the four schools, school pair 3, which accounted for 53 percent of the students in the study, outperformed the other schools in every year.

Interviews with teachers in all four schools gave us a mixed picture of the extent to which the principles of the NDP had an effect on their teaching. Some secondary school teachers who were experimenting with how to make the principles of the NDP relevant to their students appeared to be aware of ways in which they could use the students' existing knowledge in their teaching. Some teachers appeared to teach traditionally, with little regard to knowledge gained in the NDP. We were also told of uses of the NDP that seemed inefficient in that they focused on surface features of the project rather than principles. For example, one group of teachers reported that they had been advised to teach strategies for four days per week for two terms. The teachers now realised that this had not been very helpful and intended to integrate the strategies with their normal teaching in future years. Some teachers also told us that they would not be able to continue to further this integration because they had to prepare for exams in the next year.

School Pair 3

This pair of schools outperformed all other schools in each of the three years in which they were assessed. Our study set out to look at the sustainability of a mathematical concept, algebraic thinking, rather than the sustainability of the NDP. When we interviewed the teachers who were primarily involved, we found that they expressed many ideas that would be common among teachers responsible for sustaining the NDP.

We interviewed the teacher in charge of mathematics at the contributing intermediate school in 2004 and in 2006 and the HOD Mathematics at the secondary school in 2005 and in 2006. We suggest factors that may have led to their students' competence.

The important characteristics of the numeracy lead teacher in the intermediate school appeared to include a firm knowledge of relevant mathematics, of the NDP, and of the natural progression from number to algebra. She had leadership qualities that earned her the respect of other teachers and enabled her to recognise when firmer supervision of her teachers was needed. She had the full support of her principal, who was informed about the NDP. He told us "all credit went to the woman in charge of mathematics."

However, the lead teacher also gave credit to Peter Hughes, who had been their facilitator for two years. She wrote in an email "During the 2 years we were on the contract, our teaching was very focused on number and I would say it took about 80% of the [mathematics] teaching time. Also having Peter as facilitator may have contributed to our brighter students being encouraged to move from the strategies into algebraic thinking. He gave me a lot of good ideas for doing this." (Email to K. Irwin 1/5/2006.) In describing her own teaching, she said, in 2004, that she had already taken her top group into algebra. When we asked if this was traditional algebra, she appeared almost shocked. Of course not, she took the students from their use of strategies for numerical calculation into the generalities that could be expressed algebraically. This was a natural growth for her. In the email quoted above, she reported that the school had chosen algebra as one of their target areas for year 7 for 2006. "A curriculum combining algebra and number can only be a help. Very sensible." She thought that most of the teachers were managing to teach the NDP well but indicated that there were two groups of teachers that needed her help. One group was the less confident teachers. She reported that they might tell her that a certain student was perplexing them and ask for her help. She would then give that student a numeracy assessment and be able to discuss the student with the teacher. The other group that gave her some concern was teachers who had a traditional view of mathematics and were as yet unwilling to change, but she was working on this. She required all teachers to send her their planning for each group on an irregular basis and planned to do this regularly in the following year. Asked if teachers followed their projected plan, she again laughed and said that often they would still be working on what they had planned for the previous week. We formed the impression that she had a warm but professional relationship with the other teachers in the school that added to her effectiveness as a numeracy leader.

All the students were assessed at least once a year, using the NumPA or GLOSS (Global Strategy Stage) and the results passed on to the next teacher or to the secondary school. They had regular contact with the secondary schools that their students attended. They used NumPA stages as their method of reporting to parents. They had tried using asTTle (Assessment Tools for Teaching and Learning) and PAT (Progressive Achievement Test) but found the numeracy stages more useful. Her overall view was that students were enjoying mathematics more now. Their attitude was positive, and they were not afraid of using letters in place of numbers. Their understanding of the generality of the numerical operational strategies and measurement formulae for perimeter and area deepened as they explored the use of different numbers in response to "Will it work all the time?"

The HOD Mathematics at the secondary school, which these intermediate students went to, had been encouraged to participate in the SNP, but after consideration, had turned down this involvement. He knew about the study in some depth. The mathematics department had one teacher who taught only numeracy to those students who needed this. Others integrated the concepts into their traditional curriculum where they thought appropriate. In 2005, the teachers told us that they had put more emphasis on algebra than before. In 2006, the teachers in the department had taught number and algebra over terms 1 and 2 and then integrated algebra into the other topics that they taught during the rest of the year. At the interview in 2006, we told them that their students outperformed the three other schools and asked why they thought that might be. The teachers' response was that they did not think their students were very good. As the discussion developed, we learned that the HOD rewrote the departmental scheme for mathematics every year. This last year, he had incorporated some of the ideas from the Algebraic Thinking Test. In 2005, he had judged that students from intermediate schools who had had the NDP were not noticeably different from those who had not had the project. This year, his view was different. The teachers rejected the suggestion that their students had done well on this test as the result of any of their own efforts, saying that all credit must go to the intermediate school. Senior teachers at the school said that algebra was the basis of all high school mathematics and had to be brought in whenever possible. "We concentrate on algebra because they are doing badly." "Number underpins everything." They accepted different methods if the students could justify them: "Can't have one size fits all."

We formed the impression that the HOD Mathematics knew he had a difficult task in educating his students but he gave considerable thought to improving teaching methods so that his students would do better.

As demonstrated in Figure 1, the average of all their year 9 and year 10 students on the Algebraic Thinking Test had improved across the three years of the study. This could be the result of changes in teaching in the secondary school, of cohorts of intermediate students coming through with a better understanding of algebraic thinking, or a combination of both.

It may also be relevant that these two schools were relatively isolated, about an hour's drive from the nearest big city. The major influence on the intermediate numeracy leader and the head of the secondary mathematics department appeared to be the understanding and performance of their own students.

Overall Conclusions

After assessing the same students' algebraic thinking for three successive years, we can draw the following conclusions:

1. In year 9, there was no significant difference in the scores on the Algebraic Thinking Test between students from schools that had been officially in the NDP, as judged by those who put their results on the nzmaths site, and those from schools who had not put results on this site. This could be because all intermediate schools now have some understanding of the NDP, either through private providers, reading materials on the web, or though general discussions.
2. When the NumPA results of all year 9 students were correlated with the Algebraic Thinking Test results of these students, a good correlation was found. More specifically, students who reached at least the advanced additive stage in all three strategy scales were the ones who could generalise their flexibility in solving numerical problems to problems involving letters, using algebraic thinking.
3. There was a consistent increase in algebraic thinking scores over the three years in three of the four pairs of schools assessed. When results were fitted to the line of maximum likelihood, all schools showed an increase, although individual students differed.
4. One pair of schools markedly outperformed the other three pairs of schools on all three occasions. The secondary school in this pair was not part of the SNP but considered the elements in it carefully. The characteristics of the teachers in charge of numeracy and algebra at this pair of schools suggest factors that are important in transferring skills from the NDP to algebra.

Foremost in these factors are teachers with a deep understanding of the importance of algebraic thinking and what is necessary to develop it. The teachers at this pair of schools thought for themselves about the ways to help their students make this connection rather than following the advice of outside advisors. Although they saw improvement, if they were not satisfied with the results that their students were achieving, they continued to work on ways to improve students' algebraic thinking. They personified the characteristics of schools that improve their students' performance because they understood the principles of a reform, in this case the NDP. They improved their method of teaching through regular reflection on both mathematics and their students.

5. While some students did very well on the Algebraic Thinking Test, many students did not do as well as we had hoped. Factors contributing to this would include lack of appreciation of the importance of algebraic thinking for their students among intermediate school teachers and traditional teaching of algebra in secondary schools, based on rules rather than underlying concepts. Another factor would be that half of the students assessed in year 9 were not judged to have advanced additive strategies in addition, multiplication, and proportional reasoning.

This study provides some positive results but also leaves us with some concerns. We were pleased at the number of teachers who saw the relevance of algebraic thinking in the secondary mathematics curriculum, but we were also concerned about the view that, despite the importance of algebraic thinking, they could not continue to concentrate on it because of the demands of the NCEA examination in year 11. There does not appear to be a smooth transition between the curriculum for the lower secondary school years and the examination years. This does not encourage students to build on the algebraic thinking that they have developed in the NDP. This problem exemplifies the need for a smooth growth of mathematical concepts all through the school years. This should be attended to if the gains made in the primary and early secondary school years are to be developed. Algebraic thinking is essential to all secondary school mathematics but, at present, this thinking is not fostered in preparing for year 11 examinations.

In closing, we would state that the NDP has the potential to develop the algebraic thinking that underlies all secondary school algebra. However, this potential has yet to be reached for many students. Only those who are truly flexible in their use of strategies for solving additive, multiplicative, and proportional problems have gained this ability to think algebraically.

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Te Poutama Tau 2006: Trends and Patterns

Tony Trinick
The University of Auckland
Faculty of Education
<t.trinick@auckland.ac.nz>

Brendan Stevenson
Massey University
Māori Studies
<b.s.stevenson@massey.ac.nz>

This paper reports on the analysis of the 2006 data from the Māori-medium numeracy project, Te Poutama Tau. In general, student performance improved throughout 2006. However, performance on the addition, subtraction, and proportion domains was somewhat disappointing, particularly progress at years 3 and 4. Additionally, there is still a proportion of students who made minimal stage gain. Analyses of patterns of performance and progress over time from 2003 to 2006 show there have been positive longitudinal trends in most areas of the Number Framework. Students also made greater progress in the earlier stages relative to their ages. Significantly, the longitudinal trends show that, where there have been areas of concern, additional focus on these areas in subsequent years has improved performance.

Background

The New Zealand Numeracy Development Projects (NDP) were developed in response to concerns about the quality of mathematics teaching and as a result of the achievement of New Zealand students in the Third International Mathematics and Science Study (TIMMS) (Garden, 1996, 1997). Although Māori-medium kura did not participate in the TIMMS study (the study was only available in the medium of English), the Te Poutama Tau project was subsequently developed in recognition of the fact that the teaching of numeracy is a complex area and that teachers of mathematics in the medium of Māori require support. The primary aim of the Te Poutama Tau project is to improve student performance in pāngarau (mathematics) through improving the professional capability of teachers. The first Te Poutama Tau project began in 2002 as a pilot and was further extended into a range of Māori-medium kura the following year (Christensen, 2003). Te Poutama Tau is based upon the Number Framework developed for New Zealand schools (Ministry of Education, 2006a). The Framework provides a clear description of the key concepts and the progressions of learning for students. In the absence of a wide range of Māori-medium resources to assist teachers in the interpretation of the Māori-medium national curriculum statements, the Te Poutama Tau professional learning programme provides significant support for teachers who are teaching mathematics in the medium of Māori.

Teachers from 31 schools participating in Te Poutama Tau during 2006 provided data for this paper. Students were assessed individually at the beginning of the programme, using a diagnostic interview, and again at the end of the year (Ministry of Education, 2006b).

The aim of this paper is to examine the following questions:

- What overall progress did students make on the Number Framework in 2006?
- In which areas of the Framework did students perform well in 2006 and in which areas did they perform poorly in 2006? Why is this so?
- How do patterns of performance and progress of students involved in the 2006 project compare with the 2004, 2005, and 2006 patterns?
- What areas of the Framework have they performed well or poorly over the four years? Why is this so?

Method

The results for each Te Poutama Tau student, classroom, and school are entered on the national database (www.nzmaths.co.nz). The database shows the progress that students have made on the Framework between the initial and final diagnostic interview. The time between the two interviews is about 20 weeks of teaching. Schools can access their own data on the national database to establish targets for planning and reporting purposes for the subsequent year(s). Teachers can use the data to group students according to ability and use activities that will support students in both strategy and knowledge development.

Participants

The following summaries of the data were restricted to only those students with both diagnostic interview results. In 2005, 496 students completed both the initial and final diagnostic interview and in 2006, there was complete data for 1153 students.

The low number of students recorded as participating in 2005 was due essentially to a range of issues around data entry. The redesigned database at the end of 2004 made it difficult to identify Te Poutama Tau schools. If participating teachers did not enter data into the language fields, there was no easy means of identifying the participating Māori-medium kura. English-medium schools that participated in the Te Poutama Tau project also had to tick a box identifying the data as Te Poutama Tau data. A number of schools failed to do this and consequently were not identified.

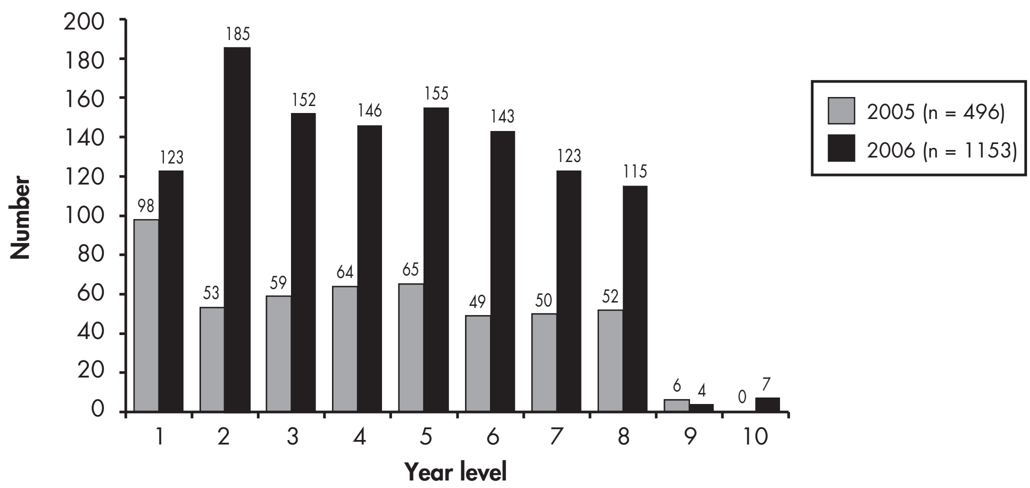


Figure 1: Distribution of Te Poutama Tau students across year levels

Overview of Student Progress 2006

Progress of students in the Te Poutama Tau schools was very positive in the areas of NID, fractions, grouping, and place value. In previous years, the results in these areas have not been as positive (Trinick & Stevenson, 2005, 2006). However, as a key component of the professional learning programme in 2005–2006, Te Poutama Tau facilitators and teachers gave particular attention to these areas. Proportion continues to be a challenge, particularly when students are in transition to stage 5 (early additive). The behavioural indicator for this stage requires students to find a unit fraction of a number mentally using addition facts, that is, $\frac{1}{3}$ of 12 as 4 + 4 + 4 = 12. The issue may be the strategy itself. This will need to be considered in future studies.

For the two forms of number-word sequencing, students make positive progress in the earlier stages, but there still appears to be an issue around the "large" numbers at stages 5 (early additive) and 6 (advanced additive), as noted in earlier studies (Trinick & Stevenson, 2005, 2006). However, there were positive stage gains in numeral identification. Hopefully, this will translate into more positive results for number sequencing.

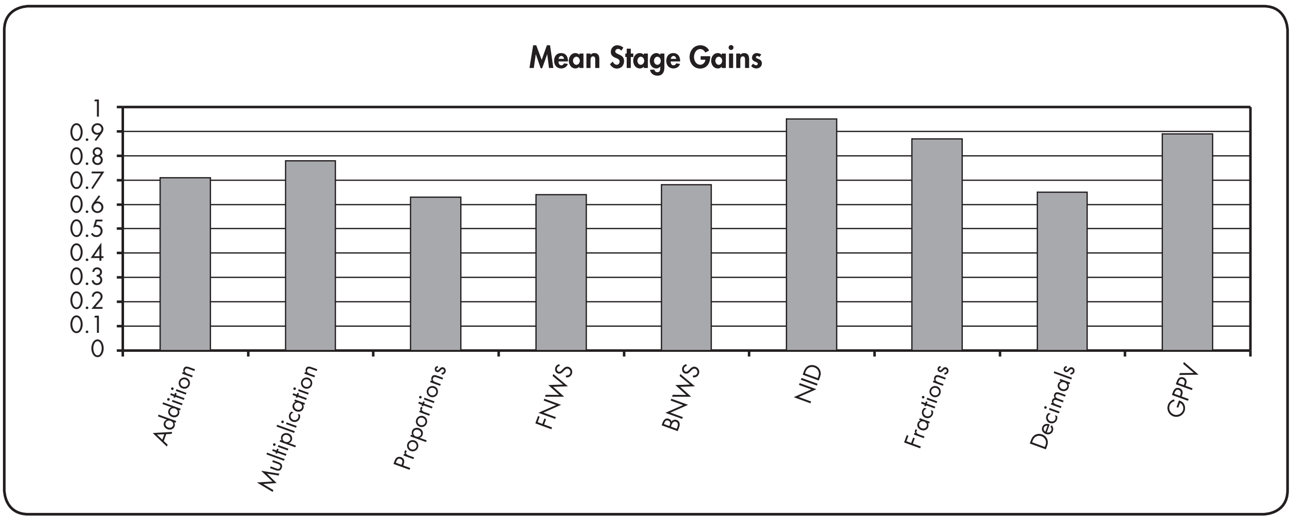


Figure 2: 2006 mean stage gains across the Number Framework

Student Achievement and Year Level

The graphs in Figure 3 on the following pages show variation in the mean gain for each domain of the Framework across the year levels. For example, students at years 0–1 made a mean stage gain of 1.25 for proportion and at year 3, a mean gain of 0.26 (Figure 3.3). There were no clear patterns common to all domains of the Framework. However, there are patterns within a number of related domains, particularly knowledge domains.

Strategy Domains

Although positive for addition and subtraction in the early stages (Figure 3.1), there is a significant slow down in later stages. This is due to a number of factors, including the complexity of upper levels and the number of students in the older year groups (6–8) who are already at stages 5–6.

There were no mean stage gains for multiplication for year 1 students and large gains for year 2 (Figure 3.2). This can be explained by the low numbers of students at years 1–2 who were tested using the multiplicative test items. The majority of years 1–2 were tested using Uiui A (NumPA) where there are no test items for multiplication. It is quite likely that the few year 2 students who made large gains were the high achievers. The results for proportion can be explained similarly. There were large stage gains at years 0–1 and at year 2 for proportion (Figure 3.3). However, there were only eight students who were tested for proportion in these year groups, and it is likely that these students may well be high achievers. There is a large dip in progress at year 3, where approximately 80 students had both initial and final data entered.

Knowledge Areas

In FNWS, BNWS, and NID, there is significant growth in the earlier years, with a similar pattern of regression in later years. This is not surprising, considering these areas are closely related. In order for students to count forwards or backwards or locate numbers, they need to be able to identify numbers. The regression can be attributed to a number of key factors. For example, a number of students in the older age groups may already be at the upper stages. It is also important to note that numeral identification (Figure 3.6) as a separate data section is only part of diagnostic interview A, so students who proceed beyond test A to tests E or U will not register mean stage progress in NID. Figure 3.6 therefore only shows progress for students who were tested using test A. NID continues to be a critical aspect in the upper stages but has been subsumed as part of ordering numbers. As already stated, in order for students to count forwards or backwards or locate numbers, they need to be able to identify numbers.

In general, there were positive results for fractions across the year groups (Figure 3.7). As noted earlier, this has been an area of focus for facilitators and teachers in the Te Poutama Tau project in 2006. However, the very positive results for the years 0–1 students can also be explained by the low numbers of students tested and the fact that they were likely to be the high achievers. In general, there were positive results across the GPPV and basic facts domains (Figure 3.8). One of the problematic areas in basic facts seems to be around the division facts at stage 7 and common factors and multiples at stage 8 (Figure 3.9).

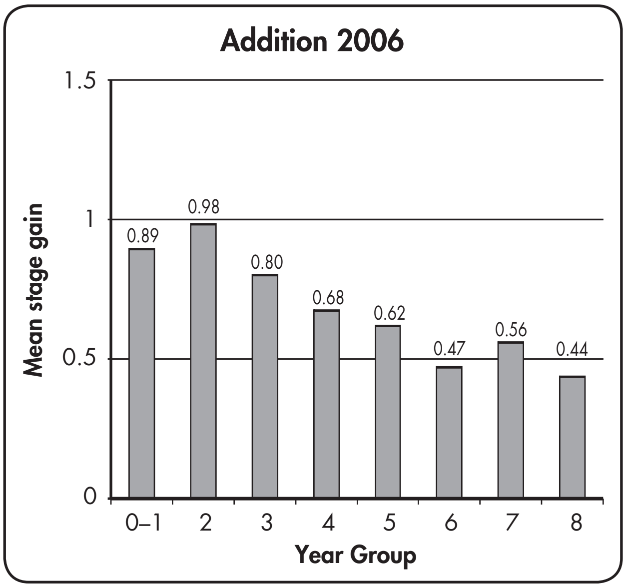


Figure 3.1: Mean stage gain for addition and subtraction

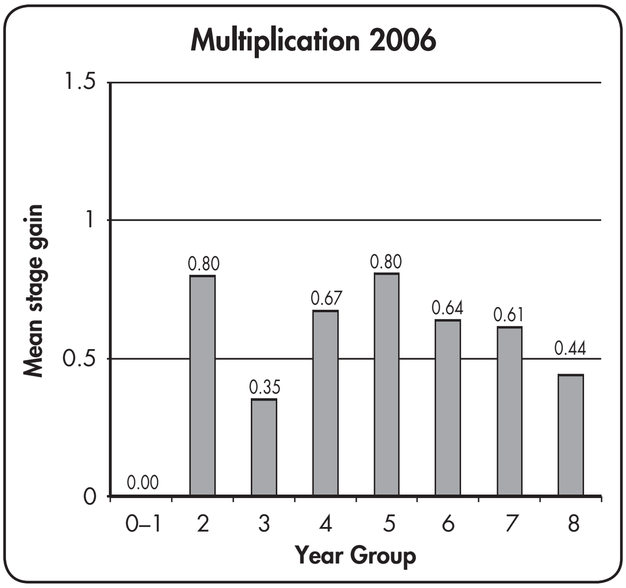


Figure 3.2: Mean stage gain for multiplication and division

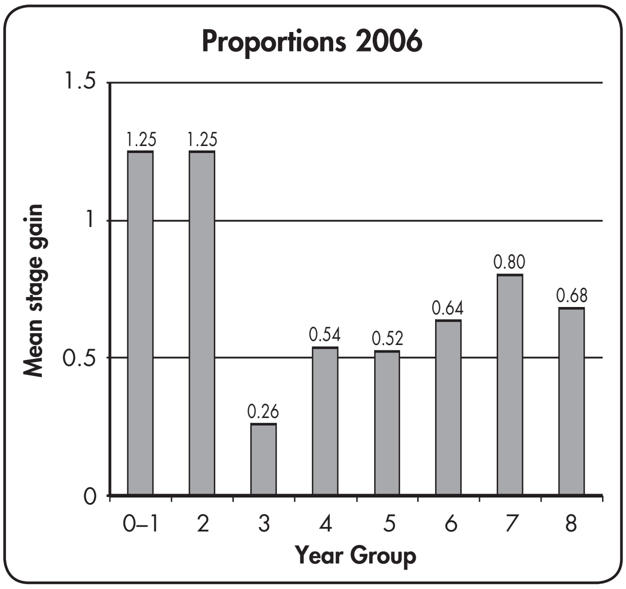


Figure 3.3: Mean stage gain for proportions

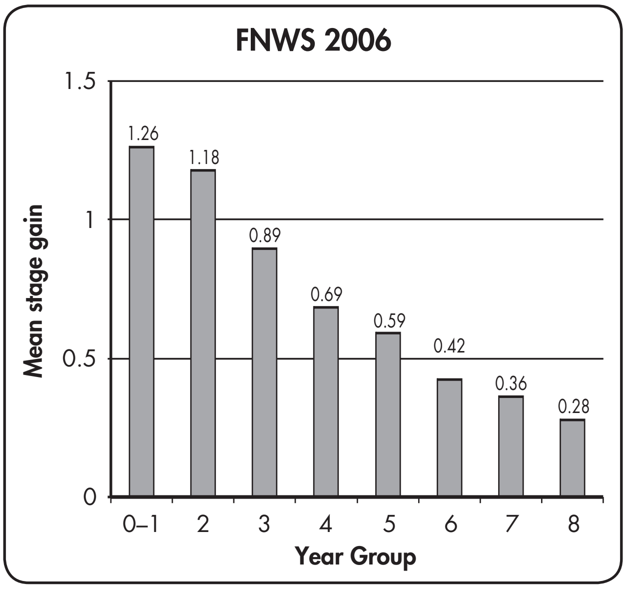


Figure 3.4: Mean stage gain for forward number word sequence

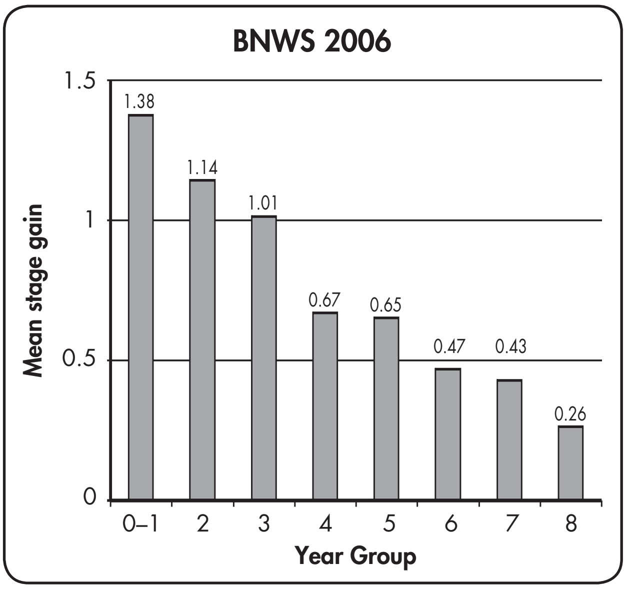


Figure 3.5: Mean stage gain for backward number word sequence

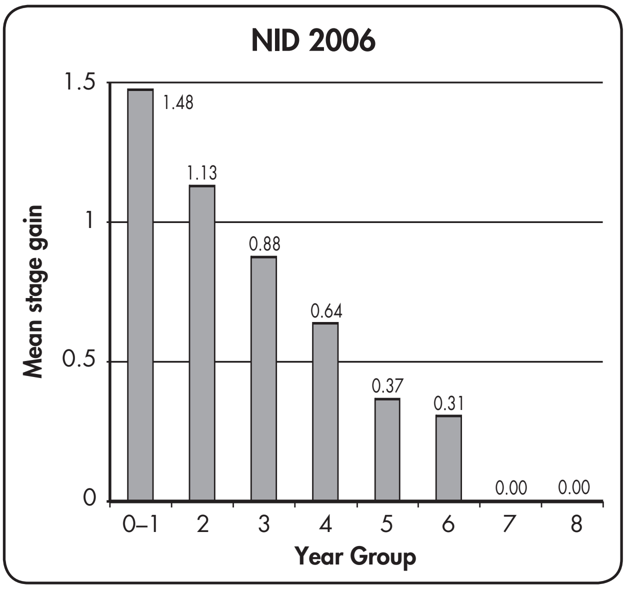


Figure 3.6: Mean stage gain for numeral identification

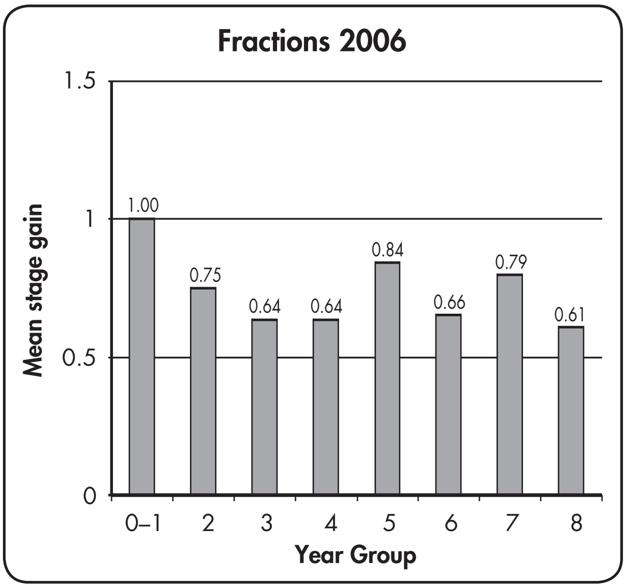


Figure 3.7: Mean stage gain for fractions

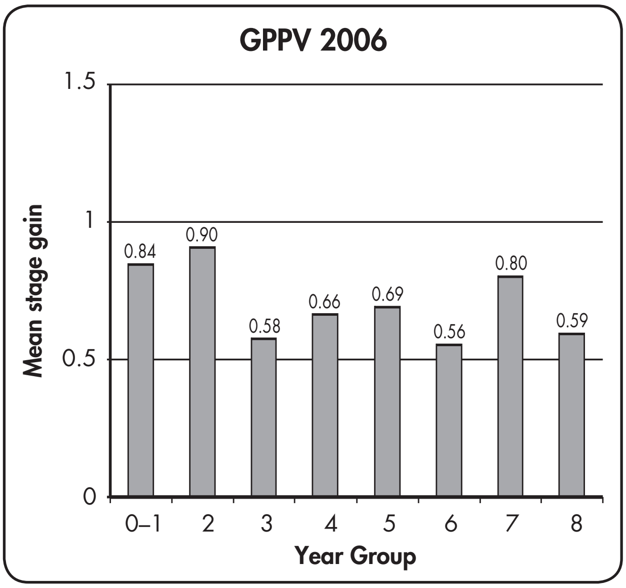


Figure 3.8: Mean stage gain for grouping and place value

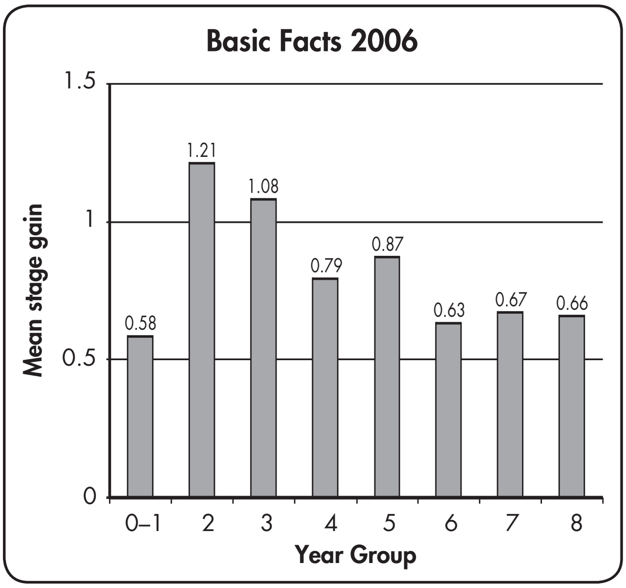
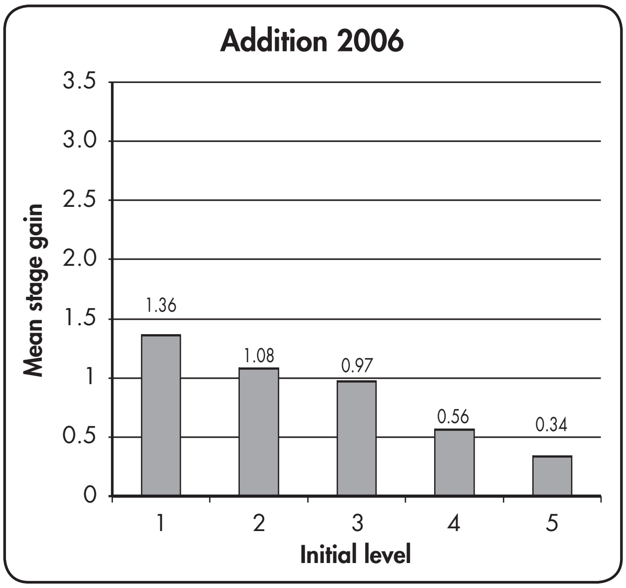


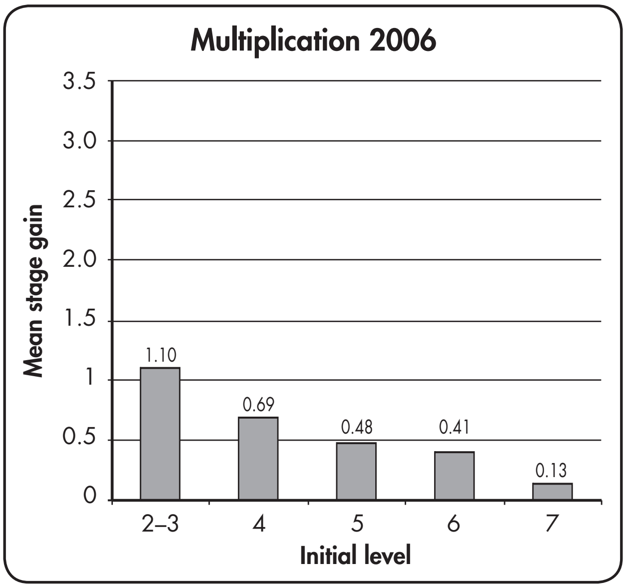
Figure 3.9: Mean stage gain for basic facts

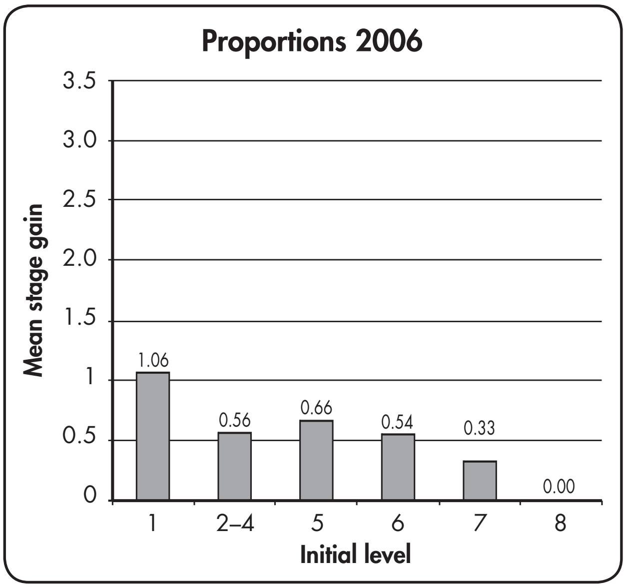
Student Achievement and Initial Stage Assessment

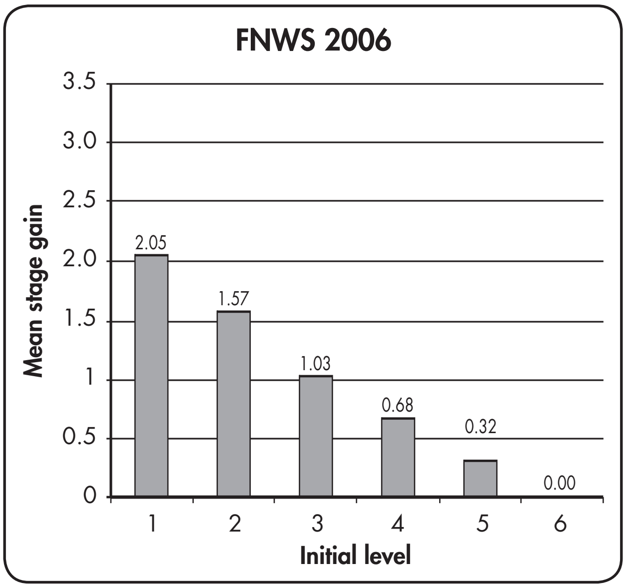
The graphs below (Figure 4) show the variation in the mean gain and initial stage level for each domain of the Number Framework. For example, students who initially tested at stage 1 for addition and subtraction made a mean stage gain of 1.36. Students who initially tested at stage 5 made a mean 0.34 stage gain. As with previous years, there was no clear pattern common to all aspects of the Framework. The domains of addition, multiplication, FNWS, BNWS, and NID showed a "diminishing returns" pattern, where advancement was more difficult for children at successively higher year levels. It is important to note that the stages on the Framework do not constitute an equal interval scale because the increments at the lower end of the Framework are smaller than those at the upper. Students tend to progress through the lower stages more quickly.

However, aspects such as fractions and GPPV are showing positive gains through most of the levels. It is particularly pleasing to note the very positive stage gains for fractions for students at stage 7.









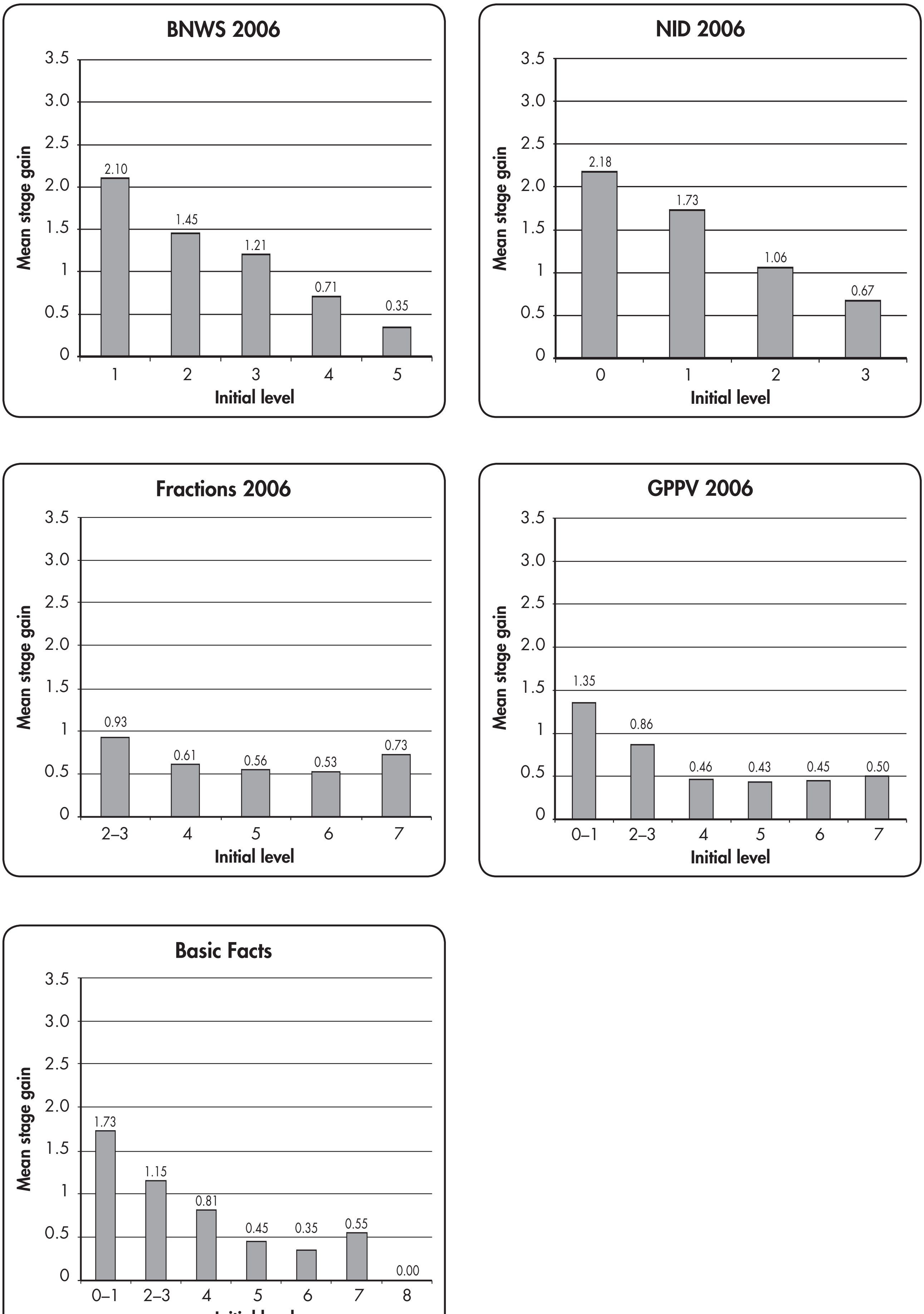


Figure 4: Mean stage gain and initial stage level

Reporting of Student Achievement Data

Individual schools and classrooms can use similar charts to analyse their own students' data. These types of charts can help teachers to identify patterns and trends at an individual school level, but teachers need to be aware that for small samples of students these charts can be very misleading. A useful addition to the NDP are guidelines for the use and reporting of student achievement data using expectations (www.nzmaths.co.nz/numeracy/Principals). This guide assists kura and teachers to identify students "at risk" and high achieving students.

Longitudinal Patterns of Progress

This section examines patterns of performance over four years of implementation of Te Poutama Tau. Overall, the trend in student progress for 2006 was relatively consistent with 2005 results. With the exception of addition and subtraction, there have been positive longitudinal trends in most areas of the Framework. One possible explanation for the regression in addition/subtraction is that some students have achieved stage 6, in other words, a ceiling affect. Over the last three years (2003, 2004, and 2005), there was evidence of improved stage gains for proportions, numeral identification, and decimals. From 2005 to 2006, there is a slight regression in fractions and multiplication. This is partly due to students moving into the higher stages, which are more complex.

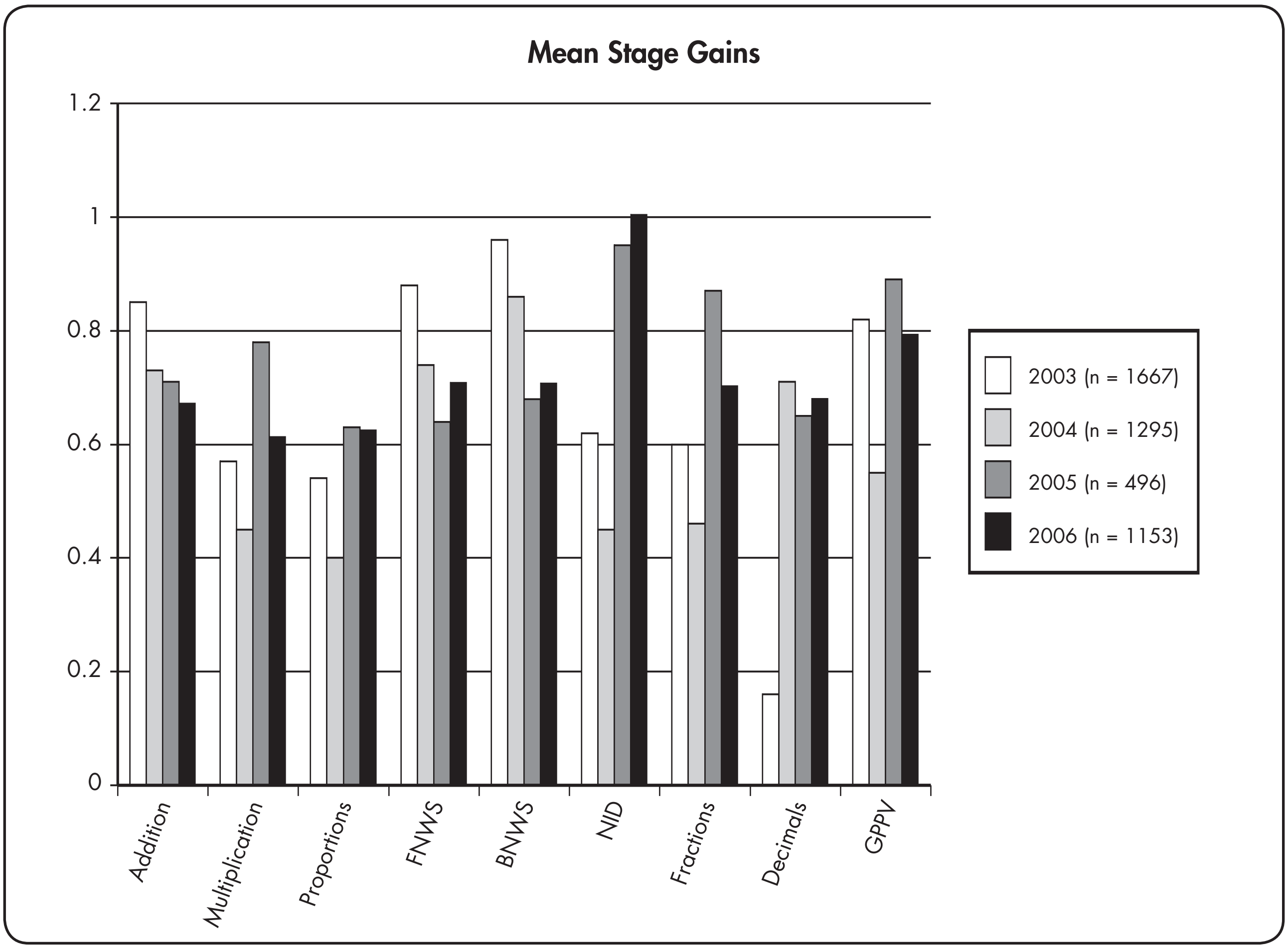


Figure 5: Mean stage gains across the Number Framework

A number of interesting trends show up in the following table. If the 2004 results are compared with the 2006 results (larger data sample sizes), the change difference in most of the domains of the Framework is greater. This in part can be attributed to increased teachers' and facilitators' confidence in the delivery and management of the project. In GPPV, for example, the trend is a 0.55 gain in 2004, a 0.89 gain in 2005, and a 0.79 gain in 2006. As noted in earlier studies (Trinick & Stevenson, 2005, 2006), grouping and place value underpin many of the key ideas of the Framework. It is not clear why there was a slight regression in addition and subtraction. As noted earlier, concern was raised in 2005 at the less than positive stage gain in NID. However, with a concentrated focus by teachers and facilitators on this domain, the mean stage gain of 1.00 is very encouraging.

Table 1
Comparison of Change Between Initial and Final Test Results

		2004 (n = 1295)			2005 (n = 427)			2006 (n = 1153)		
	Mean	Initial	Change	Final	Initial	Change	Final	Initial	Change	Final
Strategy	Addition	4.1	0.73	4.85	3.7	0.71	4.22	3.69	0.67	4.36
	Multiplication	2.1	0.45	2.58	2.6	0.78	3.16	2.63	0.61	3.25
	Proportions	2.1	0.40	2.41	2.5	0.63	2.92	2.49	0.63	3.12
Knowledge	FNWS	4.7	0.74	5.46	4.0	0.64	4.55	4.04	0.71	4.75
	BNWS	4.4	0.86	5.27	4.0	0.68	4.66	4.11	0.71	4.82
	NID	3.0	0.45	3.46	2.9	0.95	3.78	3.24	1.00	4.25
	Fractions	1.9	0.46	2.31	2.0	0.87	2.69	2.04	0.70	2.74
	Decimals	2.6	0.71	3.26	2.8	0.65	3.41	2.87	0.68	3.55
	GPPV	2.5	0.55	3.08	3.0	0.89	3.83	3.41	0.79	4.20

The following figure shows how the average for the final results for all domains varies across the year levels for 2003–2006. From year 4 onward, the trend is reasonably consistent. Large mean stage gains were made in the earlier year levels in 2006. However, as noted earlier, it is important to interpret these results cautiously because the stages do not constitute an interval scale. The large gains in the early years of 2006 can be attributed in part to the high mean stage gain in multiplication and proportions (Figure 3.3). However, the number of students who made the gains was very low. There has consistently been a dip at year 3 followed by a slight rise at year 4. It is at this point where many students are transitioning from using counting strategies to part–whole.

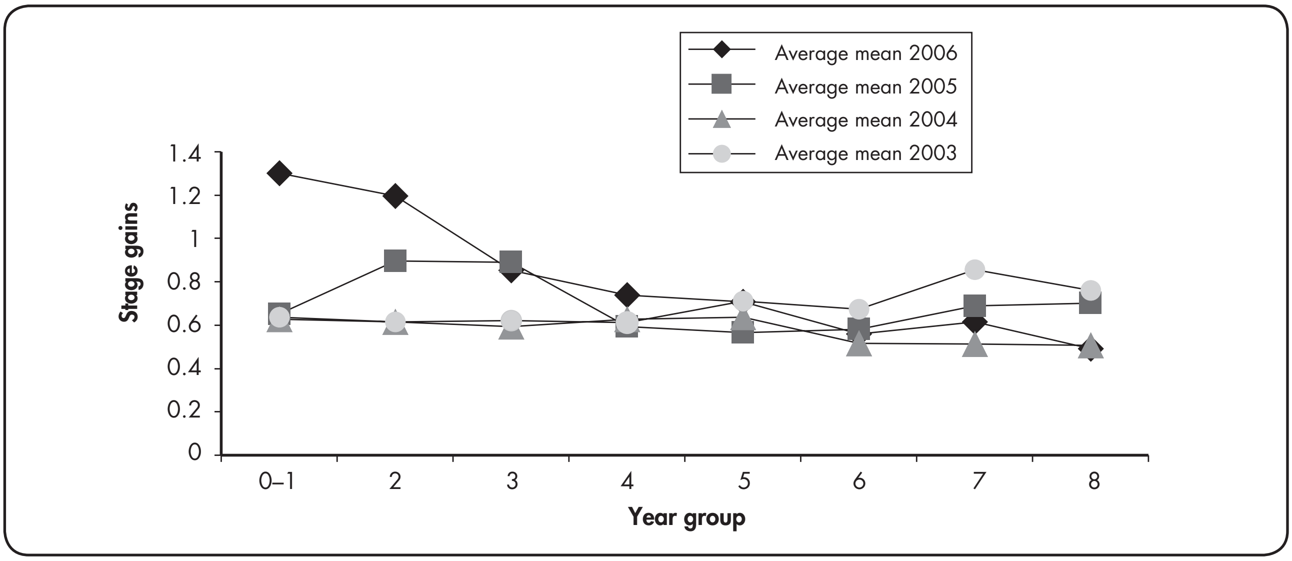


Figure 6: Comparison of students' average mean stage gain across years 2003, 2004, 2005, and 2006

Summary

As the corpus of data collected grows as a result of the Te Poutama Tau project over the last five years, there are many more questions raised, for example, the interrelationship between the domains, that is, the relationship between multiplication, division, fractions, and proportions. In order to carry out many fraction and proportion tasks, students need effective division and multiplicative strategies. Considerable work also remains in identifying the relationship between language proficiency and student achievement in Māori-medium mathematics. The following recommendations arise from the research that has been discussed in this report and discussions with Te Poutama Tau facilitators for particular focus in 2007:

- Focusing on older students who have made minimal stage gain, for example, year 4 students who have not progressed beyond the advanced counting stage for addition (these are year 5 in 2007)
- Focusing on the teaching of addition and proportion, particularly with the 2007 year 4 students
- Investigating the impact of the Te Poutama Tau project on Māori-medium mathematics generally, for example, investigating students' progress in other strands and/or using alternative tests, such as asTTLe
- Continuing to investigate the relationship between Māori language and mathematics
- Incorporating algebraic thinking into the Te Poutama Tau project. While it is unclear what the algebra objectives in the Marauatanga Pāngarau really mean for the younger students, the trickle-down effect of these objectives are clear: kura tuatahi teaching must focus greater attention on preparing all students for challenging wharekura mathematics programmes, particularly NCEA. Thus, "algebraic thinking" has become a catch-all phrase for the mathematics teaching and learning that will prepare students for successful experiences in algebra and beyond.

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"Who helps me learn mathematics, and how?": Māori Children's Perspectives

Ngārewa Hāwera
University of Waikato
<ngarewa@waikato.ac.nz>

Merilyn Taylor
University of Waikato
<meta@waikato.ac.nz>

Jenny Young-Loveridge
University of Waikato
<jenny.yl@waikato.ac.nz>

Sashi Sharma
University of Waikato
<sashi@waikato.ac.nz>

Ahakoa rongo, kāore i rongo

Ahakoa kite, kāore i kite

This study set out to explore the perspectives of Māori children attending kura kaupapa Māori schools. Forty year 5–8 children in three kura were interviewed individually in te reo Māori to ascertain their perspectives towards learning pāngarau/mathematics. The findings show that the children were aware of a number of sources of support, should they need help with their mathematics. The children had strong views about their teacher's role, strategies for learning, and working with others.

Background

In traditional Māori society, education was oral, thematic, and holistic (Barton & Fairhall, 1995; Riini & Riini, 1993). Children enjoyed the support of a variety of their community members to fulfil their potential for learning (Hemara, 2000). As educational patterns have shifted to a Western form of schooling, Māori children's underachievement in mathematics has become evident (Barton & Fairhall, 1995; Forbes, 2002; Garden, 1996, 1997; Knight, 1994; Ohia, 1995).

Initiatives have been developed and implemented to help address Māori underachievement in mathematics. These include Te Poutama Tau, a professional development programme for teachers in te reo Māori based on the English-medium Numeracy Development Projects (NDP). This programme has been implemented in some Māori immersion settings.

The views of Māori children can contribute to greater understanding about their learning in mathematics. Some Māori children in English-medium schools have provided insights regarding teacher support while learning mathematics (Taylor, Hāwera, & Young-Loveridge, 2005). However, research that considers Māori children's perspectives about learning mathematics in kura is limited. Children are major stakeholders in the business of learning in our schools, so it is important to listen to their understandings about their experiences (Forman & Ansell, 2001; McCallum, Hargreaves, & Gipps, 2000; Rudduck & Flutter, 2000; Young-Loveridge, 2005).

Children often have clear views about who supports their learning at school (Phelan, Davidson, & Cao, 1992). The roles they assign their teachers can significantly impact on their experiences during classroom mathematics sessions (Taylor, Hāwera, & Young-Loveridge, 2005). For example, if children have a view that only the teacher possesses relevant knowledge about what should be done in class, they may wait for that information to be conveyed to them (Alerby, 2003). On the other hand, children will take an active role in their mathematics learning if they perceive their teacher to be a mentor rather than a transmitter of mathematical knowledge (Taylor, Hāwera, & Young-Loveridge, 2005).

Having a range of problem-solving strategies is very helpful for children's mathematics learning (Bucholz, 2004; Thompson, 1999; Young-Loveridge, 2006). According to some writers, teachers need to take note of and help children develop their own mathematics strategies for solving problems (Heuser, 2005; Scharton, 2004; Smith, 2002). Involving children in explaining, listening to, and reflecting on a range of strategies will help them make better sense of the mathematics they engage with (Zevenbergen, Dole, & Wright, 2004).

Communication has been a major focus in mathematics learning for some time (Anderson & Little, 2004; Hunter, 2006; Ministry of Education, 1992). In order for children to gain the most from their learning in mathematics, they need to have meaningful interactions with those around them (Ittigson, 2002; Lyle, 2000). However, expectations may need to be made explicit to children so that they appreciate the value and purpose of such interactions (Campbell, Smith, Boulton-Lewis, Brownlee, Burnett, Carrington, & Purdie, 2001; Hunter, 2006). According to Christensen (2004), student discussions in pāngarau/mathematics in Te Poutama Tau classrooms have tended to be limited to short responses to recall questions involving calculations.

Close relationships with others in class may affect Māori children's participation and learning (Bishop & Berryman, 2006; Bishop, Berryman, Tiakiwai, & Richardson, 2003; Macfarlane, 2004). Working co-operatively with others has long been deemed a useful strategy for learners of mathematics (Terwel, 2003; Kumpulainen & Kaartinen, 2004). Tasks that require co-operative learning and the social construction of mathematics ideas are thought to be helpful for Māori (Hāwera, 2006; Holt, 2001). An integral part of this is positive interdependence, where participants perceive that common goals can only be achieved when all members attain their personal goals. Such a process encourages the sharing and justifying of ideas and the resolution of conflicting perspectives and solutions and hence stimulates higher cognitive processing (Johnson & Johnson, 1999).

Although there is considerable research on children's views of their learning at school, there is a paucity of information about children's perspectives regarding whānau/family support for their learning of pāngarau/mathematics. Atkinson (1999) suggests that parents who wish to support their children in schools may need exposure to recent developments in order to work with teachers and children to raise mathematics achievement. Te Poutama Tau emphasises mental calculation and a range of non-algorithmic strategies in number activities. Such emphases may be different from those learned by parents and extended whānau.

The purpose of this study was to explore the views of Māori children attending kura kaupapa Māori schools or wharekura about their perceptions of the support they receive when learning mathematics.

Method

Participants

This study focuses on the responses of 40 year 5–8 Māori children in three schools. Two schools were kura kaupapa Māori, catering for students from years 0 to 8, and one was a wharekura with students from years 0 to 13. All kura had participated in Te Poutama Tau, the Māori immersion component of the NDP, for several years prior to the study. Half of the children were from a decile 1 kura, and half were from decile 5. Twenty-three of the children were female and 17 were male. Table 1 shows the composition of the sample by year level and highest Framework stage on Te Mahere Tau (The Number Framework; see Ministry of Education, 2007) in mid 2006.

Table 1
Composition of the Sample by Year Level and Highest Framework Stage

Year level	Yr 5	Yr 6	Yr 7	Yr 8	Total
Highest Framework stage					
3	1				1
4	1	1			2
5	4	2		3	9
6	2	3		5	10
7		2	10	4	16
8		1	1		2
Total number of children	8	9	11	12	40

Procedure

Schools were asked to nominate year 5–8 children from across a range of mathematics levels. Children were interviewed individually for about 30 minutes in te reo Māori in a quiet place away from the classroom. They were told that the interviewer was interested in finding out about their thoughts regarding their learning of pāngarau/mathematics.

The questions this paper focuses on were part of a larger collection of questions that the children were asked to respond to. The questions of interest here were:

- Ki ōu whakaaro, he aha ngā mahi ā tō kaiako hei āwhina i a koe ki te ako pāngarau? (How do you think your teacher helps you to learn mathematics?)
- Pēhea ētehi atu tāngata? Ka āwhina rātou i a koe ki te ako pāngarau? Ko wai? Pēhea? (What about other people? Do they help you to learn mathematics? Who? How?)
- Kei te kāinga ētehi tāngata hei āwhina i a koe ki te ako pāngarau? Pēhea tō rātou āwhina? (Are there people at home who help you to learn mathematics? How do they help?)
- He aha tō hiahia i te nuinga o te wā – me mahi ko koe anahe, me mahi rānei ki te taha o ōu hoa? He aha ai? (How do you prefer to work most of the time – by yourself or with your friends?)

Audiotapes of interviews were transcribed by a person fluent in te reo Māori. Transcripts were subjected to a content analysis to identify common ideas coming through in the children's responses.

Results

Children's responses to the questions have been organised according to the various themes emerging from the data. Some examples illustrating the range of responses have been recorded below. The code at the end of each excerpt identifies the child as well as gender and year level.

Teacher's Role

The children were asked how their teacher helps them to learn mathematics: "Ki ōu whakaaro, he aha ngā mahi ā to kaiako hei āwhina i a koe ki te ako pāngarau?"

The most common response from children was to refer to strategies that their teacher had taught them (see Table 2).

Table 2
Children's Views Regarding Teacher Help

Shows strategies	Mathematics skills	When difficult	Teacher's behaviour	No help	No idea
16	3	3	8	1	9

Shows strategies

Sixteen of the 40 children mentioned that the teacher helped mostly by showing them a strategy or strategies to do the mathematics. These children placed great reliance on the teacher to supply them with the way/ways to do the mathematics:

> Ka mahi ia tētahi pātai pāngarau i runga i te papatuhituhi. Ana, ka pātai ia ki a mātou pēhea ka mahi tētahi rautaki mō tēnei whakautu. Ara, ka tarai mātou, ara, ka tuhi ia tētahi rautaki kia mārama mātou ki tētahi rautaki rerekē mō taua pātai. Āe. (K4–f7)
>
> (He does some mathematics questions on the board, and he asks us how would we use a strategy for this answer. We try, and he writes another strategy so that we can understand a different strategy for that question.)
>
> A, ka whakaatu mai ia i ētahi rautaki kia māmā ake te haere mō te pāngarau, ... kore tahi noa iho, āhua toru, āe, āe. (K38–f7)
>
> (He shows us some strategies so that the mathematics is easier ... not just one, about three, yes, yes.)

Mathematics skills

Three of the children mentioned that their teacher helped them to develop particular mathematics skills:

> ... ki te kaute i ōku nama" (K15–m5)
>
> (... to count my numbers)
>
> Ina kāre koe i te mōhio i te rua whakarau rua, ka whakaako ia. (K29–f6)
>
> (If you don't know 2 x 2, he will teach you.)

When mathematics is difficult

Three others mentioned that the teacher helps when the mathematics is "difficult":

> ... ka āwhina a ia i a koe mēnā ka ngaro koe (K21–f5)
>
> (... helps us if we get lost)
>
> ... ka taea e ia ki te āwhina i a mātou i ētahi wā, mēnā e uaua te pātai (K13–f8)
>
> (...helps us sometimes when the question is difficult)
>
> ... kia mahi mai i ngā mea māmā ki ngā mea uaua (K45–m5)
>
> (...helps do the easy-to-difficult ones)

Teacher's behaviour

Eight children commented on the teacher's behaviour. The teacher was described as someone who:

> kōrero ngātahi (K23–f7)
>
> (talks with us)
>
> ki te whakamārama i ngā, he aha mātou me mahi (K21–f5)
>
> (explains what work we have to do)
>
> te tuhi i runga i te papa tuhituhi (K20–m6)
>
> (performs tasks like "writing on the board")
>
> mahi i ngā mea uaua ake mōkū (K39–m6)
>
> (provides me with "harder work")
>
> ka tohatoha ngā kurū, ngā hints, āe, ki a mātou ... (K35–m7)
>
> (shares clues and hints, yes, to us ...)

Teacher is no help

One child was adamant that the teacher did not help at all in his learning of mathematics (K14–m8).

No idea about the teacher's role

Nine of the children did not seem to have any view about how their teacher helped them with their mathematics learning. The idea of thinking about and discussing the role their teacher plays in their mathematics learning seemed to be something they had not previously considered.

Support from Friends

The children were asked about other people, whether or not they help them learn mathematics, and how: "Pēhea ētehi atu tāngata? Ka āwhina rātou i a koe ki te ako pāngarau? Ko wai? Pēhea?"

This gave them an opportunity to reflect on the contribution of their friends or peers.

Table 3
Children's Views Regarding Help from Others

Help from friends	No help from friends	No mention of help from friends
24	9	7

Help from friends

Twenty-four children mentioned that others in their class helped them.

Eleven of these 24 children said that their friends helped by showing them a strategy or a way to do their mathematics:

> Ka whāki mai rātou pēhea te mahi. (K29–f6)
>
> (They reveal to me how to do the work.)
>
> Ka kōrero mai rātou he aha tātahi rautaki pai ake. (K19–f8)
>
> (They tell me a better strategy.)

Four people viewed friends as peers who provided them with an answer:

> Ka kī mai i ngā whakautu. (K36–m7)
>
> (They tell me the answer.)
>
> ... te kī mai i ngā whakautu ... kāore i te pai, nā te mea e pīrangi ana au kia ako (K18–f8)
>
> (... tell me the answers ... not good because I want to learn)

Three saw friends as people who were able to explain the work to them:

> Mēnā kāre au i te mārama ētahi wā ka whakamārama rātou ki ahau. (K25–f7)
>
> (If I don't understand, they will explain it to me.)

Four children saw friends as people they could work with:

> Ka āwhina mātou katoa i a mātou. (K26–f8)
>
> (We all help each other.)

Two of the 24 were not specific about how their friends helped.

No help from friends

Nine out of 40 children stated specifically that they received no help from their peers with their mathematics learning. In fact, five of these children were very clear in their view that they were so strong mathematically compared to others in their class that it was their peer group who expected help from them, rather than the other way round:

> Ētahi wā ka whai rātou i ōku mahi. (K38–f7)
>
> (Sometimes they follow my work.)
>
> Kāo, ka hiahia rātou i ahau ki te whakaako i a rātou. (K40–f5)
>
> (No, they want me to teach them.)
>
> Ka whai rātou i ahau. (K17–f6)
>
> (They follow me.)

No mention of help from friends

Seven of the 40 children made no specific mention of friends at school helping them with their mathematics.

Support from People at Home

Another question that children were asked to respond to was about people at home who help them learn mathematics, and how they help them learn: "Kei te kāinga ētehi tāngata hei āwhina i a koe ki te ako pāngarau? Pēhea tō rātou āwhina?"

Table 4
Children's Views Regarding Help at Home

Strategies	Mathematics skills	Questions	Various ways	Not sure how	No help
9	8	8	9	5	1

Thirty-nine out of the 40 children interviewed responded immediately that there were people at home who help them with their mathematics learning. These included mothers, father, grandparents, siblings, as well as uncles and aunties.

Strategies

Nine of these children commented on how people at home helped them with strategies to learn.

> Ka homai rātou te rautaki kia māmā ake. (K29–f6)
>
> (They give me the strategy so that it's easier.)
>
> Kāre rātou ka kī te whakautu, ka kī rātou ētahi rautaki mōku, āe. (K38–f7)
>
> (They don't tell me the answer, they tell me some strategies.)

Mathematics skills

Eight out of the 40 children were quite specific about the mathematics that those at home helped them with:

> Ka kore au e mārama i ngā mahi tau ā ira ... ka whakahoki ētahi mahi kāinga, ka āwhina rātou i a au. (K27–f8)
>
> (If I don't understand decimals ... I take home some homework, they help me.)
>
> Ki te kaute me ahau, and ki te whakaako i ahau he aha ngā tangohia me ngā whakarea me ngā honohono. (K21–f5)
>
> (To count ... to teach me subtraction, multiplication, and addition)

Questions

Eight children mentioned being asked to answer questions:

> Ka whiu pātai ki au. (K16–f6)
>
> (They ask me questions.)
>
> Ka whakaatu ia ētahi pātai, ā, ka whakautu au, and mēnā kāre he tika me haere tonu au kia whiwhi i te mea. (K42–m7)
>
> (She shows me a question, I answer it, and if it's not right, we keep going until we get the one.)
>
> A, ia rā whānau ka kī a ia, ka hoatu au ki a koe rima tekau tāra, mēnā ka taea koe te mahi i ēnei pātai tahi rau i roto i tēnei rā ... tino uaua, arā, ka awhi i ahau. (K39–m6)
>
> (On each birthday, he gives me $50 if I can answer 100 questions on that day ... very difficult and he helps me.)

Other

Two children talked about family members who gave them clues but not the answers:

> ... ka whoatu i ngā hints (K35–m7)
>
> (... gives hints)
>
> Mā te kī ko tēhea te nama tata ki te mea tika (K20–m6)
>
> (By saying the number close to the right one)

Five felt that they were given help generally with their homework. Two children mentioned that there was help at home for them, but they didn't use it.

Not sure how they helped

Five children were not sure how people at home helped them learn mathematics.

No help at home

Only one child said there was no one at home to help her.

Preferred Way to Work

Later during the interview, the children were asked how they preferred to work most of the time, by themselves or with friends: "He aha tō hiahia i te nuingā o te wā – me mahi ko koe anahe, me mahi rānei ki te taha o ōu hoa? He aha ai?"

Table 5
Children's Preferences Regarding Working Alone or with Friends

Always work with friends	Work alone except for difficult ones	Always work alone
16	8	16

Sixteen children out of 40 indicated that they would prefer always to work with their friends. Fourteen of these thought that this would be helpful for their own learning:

> Nā te mea ka taea rātou ki te āwhina i ahau (K11–f5)
>
> (Because they can help me)
>
> Ka taea koe te ako. (K23–f7)
>
> (You can learn.)
>
> He māmā ake. (K37–f7)
>
> (It's easier.)

The other two children felt that learning maths with others was helpful for their friends rather than for themselves:

> Kia mohio hoki ō hoa ki ngā whakautu (K24–m7)
>
> (So that your friends will know the answer)
>
> Kia pai ake, kia tūturu ōna mōhiotanga (K13–f8)
>
> (So that his/her knowledge is better and more secure)

Help with Challenging Mathematics Only

Eight children thought working with others was useful but only when working on "harder" or more difficult mathematics; otherwise it was better to work alone:

> Um, mēnā he tino uaua te pātai, ka haere ki tētahi o ōku hoa ki te mahi rautaki, āe, mēnā he māmā ngā mea katoa, āe, mahi ko koe anake. (K38–f7)
>
> (If it's a difficult question, I'll go to one of my friends to work on a strategy. Yes, if it's all easy, work by yourself.)
>
> Mēnā kāore koe e mōhio pēwhea te mahi pāngarau, taea te mahi tāu hoa taha, āe, mēnā e koi rawa koe, āe, taea te mahi tō ake taha. (K31–m8)
>
> (If you don't know how to do the mathematics, you're able to work with your friend. Yes, if you're really sharp, you can work by yourself.)

Sixteen out of the 40 children stated that they always liked to work alone, and a variety of reasons were given. Five felt that their friends talk too much:

> Mahi ko au anake ... ka kōrero rātou. (K14–m8)
>
> (Work by myself ... they talk.)

Five thought that their friends would "copy" or "steal their answers":

> Kia kore ia ka titiro ki ō mahi pāngarau, me te tinihanga (K15–m5)
>
> (So they don't look at your work and cheat)

Another five felt that there were other advantages to working independently:

> Kia taea ki te eke ki tērā taumata (K35–m7)
>
> (So that I can get to the next level)
>
> E pīrangi ana au kia mahi ko au anake, kia kore au e bored. (K18–f8)
>
> (I like to work alone so that I don't get bored.)
>
> Nā te mea, ētahi wā he āhua rerekē ngā whakautu, arā, ka whakamahi i taua whakautu engari ka hē, arā, he tika tōku, koirā te take ka mahi au ōku ake, nā te mea ina he hē, he pai. (K26–f8)
>
> (Sometimes the answers are a bit different, and when I use that answer it's wrong and mine was right. That's why I like to work by myself, because if it's wrong, that's OK.)

Only one of these children could not articulate a reason for preferring to work alone.

Discussion

It was pleasing to see from the children's responses that they were aware of a number of sources available to them, should they require support for their mathematics learning. Most children thought that there was help readily available for their mathematics learning, from their teachers, their friends, and/or their families.

Many children indicated that the teacher played an integral part in their mathematics learning by providing them with particular strategies and help when they were experiencing difficulty. The children seemed to regard their teacher as the person who was responsible for controlling and determining their mathematics programme. Their responses indicated that they thought very little input was required of them. Could this perception of the teacher's role and the consequent modes of participation by the children impose some limits upon children's mathematics learning?

Te Poutama Tau emphasises the need for children to learn a range of strategies to support the development of number ideas. The idea that there are different and acceptable ways of finding a solution was clear to these children. However, there was little evidence to suggest that children were being encouraged to generate mathematics ideas or strategies of their own (Heuser, 2005; Scharton, 2004; Smith, 2002). Communication with the teacher or peer group seemed to be restricted to explanations of strategies that had originated from the teacher. Like Christensen (2004), this study found that interactions in pāngarau/mathematics did not seem to involve the children in major discussions about key mathematical ideas.

Although the children had learned that there can be multiple strategies to reach solutions, none of them mentioned the possibility that these strategies could be the basis for in-depth problem-solving or investigative work that was academically engaging and mathematically challenging (Bastow, Hughes, Kissane, & Mortlock, 1984; Colomb & Kennedy, 2005; Maxwell, 2001; Ministry of Education, 1992; Terwel, 2003). It is clear that open-ended tasks that appeal to children's different experiences and levels of thinking are important (Ittigson, 2002; Terwel, 2003). According to *Mathematics in the New Zealand Curriculum* (MiNZC: Ministry of Education, 1992), such open-ended problems place more emphasis on the process of problem solving and require persistent and sustained engagement over a period of time (Bastow et al., 1984; Colomb & Kennedy, 2005; Maxwell, 2001). This approach to mathematics has been shown to be beneficial for Māori learners (Hāwera, 2006; Hemara, 2000).

In recent years, there has been much emphasis on mathematics learning as a social activity (Ernest, 1994; Hunter, 2006; Ittigson, 2002; Ministry of Education, 1992). However, the benefits of working co-operatively or collaboratively in mathematics (Terwel, 2003; Johnson & Johnson, 1999; Kumpulainen & Kaartinen, 2004) were not always apparent to these children. Although more than half of them thought that it could be helpful to work with their friends, many expressed a strong preference for working by themselves on mathematics tasks for fear of distraction, being cheated on, or their individual progress being hampered. Some children recognised the advantages of collaboration when the mathematics was more challenging, wanting to share the responsibility for solving problems set by the teacher. Hunter (2006) argues that the benefits of working together should be made more explicit to children if they are to value co-operative and collaborative mathematical experiences at school. This is consistent with the notion of mathematics as a social activity and with Māori concepts of ako (reciprocal learning and teaching) and whānaungatanga (relationships) that enhance learning for Māori (Macfarlane, 2004). However, it is important to remember that the practice of discussing, reasoning, and playing with ideas when learning mathematics is not equally "natural" for all students (Lubienski, 2007). Teachers need to be aware that some students may need more support than others in adopting discussion-based approaches to their mathematics learning.

It was overwhelmingly clear that these children were aware of having strong support at home to help with their mathematics learning. This support involved giving children strategies, answering questions, and clarifying particular mathematics ideas. There was no evidence of conflict between the learning of particular mathematics strategies at school and the support that was available at home. This could indicate that the children have become accustomed to the idea that there can be more than one way to find a solution to a mathematics question and fully accept that notion. Families clearly have a powerful influence on children's learning. Could more opportunities be created to take advantage of this support to help address underachievement of Māori in mathematics? This might involve sharing recent initiatives and emphases in mathematics learning with whānau, thereby helping to address a key aspect of the NDP strategy; that is, strengthening links with the community (see Ministry of Education, 2001).

This study indicates that these Māori children participating in Te Poutama Tau think they have considerable support from teachers, friends, and whānau with their mathematics learning, should they want it. Teacher-taught strategies were viewed as the ultimate authority in the mathematics programme. Despite the emphasis on listening to and building on others' ideas in Te Poutama Tau, the children seemed to have few expectations that they needed to contribute to the construction of their own mathematics ideas. Many also seemed unaware of the possible benefits of collaborative learning, even though this has been a successful strategy used by Māori in earlier times (see Hemara, 2000).

Recommendations

This study has raised issues for educators of Māori children. Improving the mathematics achievement of Māori children is an ongoing focus. We suggest that the following ideas be considered:

- more exploration and development of ideas by children to enhance their ability to make sense of mathematics
- help for children to participate in and appreciate demanding mathematical discourse
- inclusion of more challenging problem-solving and investigative tasks
- utilising and building upon children's ideas for their mathematics programme
- ensuring that tasks requiring collaboration are included in mathematics programmes
- creation of more opportunities for the use of the strong whānau support available for mathematics learning
- further research to explore ways of continuing to enhance mathematics learning for Māori children.

Ngā Mihi

Hei whakamutu ake tēnei wāhanga o te rangahau, ka mihi ake ki ngā whānau, ngā mātua, ngā tamariki i whakaae kia uru mai ki tēnei rangahau. Mā te mahi pēnei ka mārama pai ai te huarahi, ka hiato ngā whakatupuranga.

Nō reira, ngā karanga maha, ka nui te mihi.

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Year 7–8 Students' Solution Strategies for a Task Involving Addition of Unlike Fractions

Jenny Young-Loveridge
University of Waikato
<jenny.yl@waikato.ac.nz>

Merilyn Taylor
University of Waikato
<meta@waikato.ac.nz>

Ngārewa Hāwera
University of Waikato
<ngarewa@waikato.ac.nz>

Sashi Sharma
University of Waikato
<sashi@waikato.ac.nz>

This study reports on data gathered from 238 year 7–8 students in six intermediate schools who were given a task involving addition with fractions ($\frac{3}{4} + \frac{7}{8}$). Only 32 students (13%) found a correct answer for the problem, and some of those solved it using procedural knowledge rather than a deep conceptual understanding of fractions. Half of the students gave an incorrect answer. The most common error, shown by almost a quarter of the students, was to add the numerators and/or denominators (the "add across" error). More than a third of the students did not attempt the problem. Students' difficulties are analysed and the implications of the findings for teachers discussed. The potential value of the "make a whole" strategy for helping students understand about the properties of fractional numbers is considered.

Fractional or rational numbers are important in everyday-life situations (Anthony & Walshaw, in press), allowing us to answer questions not just about "how many" but also about "how much". Even preschool children use fractions when trying to determine "fair shares". Real-life measurement problems often require an understanding of rational numbers if precise measurements are to be made. Many everyday activities, such as shopping, rely on an understanding of rates such as price per litre or price per kilogram. Speed limits are presented as the relationship between distance and time (for example, 50 kilometres per hour). Percentages, a particular type of fractional number, are important to anyone with a mortgage or a bank loan. Discounts are usually presented as percentage reductions in price.

Learning about fractions presents considerable challenges for students throughout their school years (Anthony & Walshaw, in press; Behr, Lesh, Post, & Silver, 1983; Brown & Quinn, 2006; Charalambous & Pitta-Pantazi, 2005; Davis, Hunting & Pearn, 1993; Empson, 1999, 2003; Hunting 1994, Lamon, 2007; Pearn & Stephens, 2004; Smith, 2002; Usiskin, 2007; van de Walle, 2004; Verschaffel, Greer, & Torbeyns, 2006). The difficulties that students experience with fractions can cause problems with other domains in mathematics such as algebra, measurement, and ratio and proportion concepts (Behr et al., 1983; Lamon, 2007; van de Walle, 2004). On the other hand, teaching students how to abstract mathematical ideas in the context of fractions can be extremely beneficial to their algebra learning (Wu, 2002).

Although fractions are known to be difficult to teach and learn, they have been described as one of the most "mathematically rich" and "cognitively complicated" areas of primary school mathematics (Smith, 2002). Moreover, they "are among the most complex and important mathematical ideas children encounter during their pre-secondary school years" (Behr et al., 1983, p. 91). It seems likely that the difficulties that students experience with fractions are related to their complexity. Various frameworks have been proposed to account for the different ways that fractions can be interpreted, including Kieren's system of five sub-constructs (see Behr et al., 1983). Understanding fractions requires an understanding of each of the sub-constructs as well as the ways in which the sub-constructs are connected. Arguably the most important sub-construct, the one underpinning all other sub-constructs, is the *part–whole* or *partitioning* sub-construct. However, unlike the partitioning that occurs with the addition and subtraction of whole numbers (which can be of unequal parts), partitioning for fractions (as well as for multiplication and division) must be of equal-sized parts (see Pothier & Sawada, 1983). For this reason, *equivalence* is a key aspect of the part–whole sub-construct.

Kieran's other four fraction sub-constructs include *ratio*, the idea of relative magnitude, necessary for understanding ideas about proportion and equivalence (as in renaming $\frac{3}{4}$ as $\frac{6}{8}$, $\frac{9}{12}$, $\frac{15}{20}$, and so on); *operator*, necessary for the multiplication of fractions (as in $\frac{3}{4}$ of 10 metres); *quotient*, necessary for problem solving (as in $\frac{1}{4}$ of 20 means the division of 20 by 4); and *measure*, necessary for addition of fractions (as in $\frac{3}{4}$ is the same as $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$). Another way of looking at fractions is to see them as both a *process* (for example, $\frac{3}{4}$ involves division of 3 by 4) and a *product* that results from a process ($\frac{3}{4}$ is the result of dividing 3 by 4) (see Verschaffel, Greer, & Torbeyns, 2006). The dual meaning of mathematical symbolism as *processes* to *do* and *concepts* to *know* has been captured in the term "procept" (Gray, Tall, & Pitta, 2000). Students can confuse fractions as *numbers* (that is, single quantities, as in $\frac{3}{4}$ of a litre) and fractions as *operations* (that is, proportions of quantities, as in $\frac{3}{4}$ of 10 metres). Physical materials can be used to model fraction tasks; different ways of modelling fractions include *region* or area models, *length* or measurement models, and *set* or collections models (van de Walle, 2004). Both region and length models involve *continuous* quantity, whereas set models involve *discrete* quantity. It has been suggested that one way to deal with all of this complexity is to define a fraction as simply "a point on a number line", allowing these different meanings of fractions to be deduced using logical reasoning (Wu, 2002). However, that has the disadvantage of excluding other powerful ways of coming to understand fractions using regions or sets.

Recent literature supports the idea that multiplicative thinking is essential for a deep and connected understanding of fractions, including proportions (Lamon, 2007; Thompson & Saldhana, 2003). It requires the recognition that "times" means "to envision something in a particular way – to think of copies (including parts of copies) of some amount" (Thompson & Saldhana, 2003, p. 104). It involves the realisation that there is an important reciprocal relationship, so that if quantity X is $\frac{1}{n}$ of a quantity Y, then Y is n times as large as X. Those who interpret fractions as "so many out of so many" (as in $\frac{1}{n}$ is "one out of n parts") are thinking of fractions *additively* instead of *multiplicatively* and will have difficulty dealing with situations where one quantity's size is a fraction of another quantity's size, when the quantities have nothing physically in common (for example, "the number of boys is what fraction of the number of girls?"). Hence, understanding *5 x 4* multiplicatively requires the understanding that the *4* in *5 x 4* is not just 4 ones (as in 20 = 4 + 9 + 7), but that the *4* is special because it is $\frac{1}{5}$ of the product (Thompson & Saldhana, 2003).

There is some debate in the literature about whether or not the teaching of algorithms is a good idea (Kamii & Dominick, 1998; Lappan & Bouck, 1998; Wu, 2002). Opponents of algorithms argue that some children never learn the algorithm and that those who can carry out algorithms don't always understand why or how they work, so they have little sense of when an algorithm is useful for solving a problem (Kamii & Dominick, 1998; Lappan & Bouck, 1998). They object to algorithms on the grounds that being told exactly how to do something "encourages children to give up their own thinking" (Wu, 1999, p. 4). Lappan and Bouck (1998) advocate the use of complex problems that encourage students to invent their own algorithms for adding and subtracting fractions. They argue that, although it takes more time to let students "wrestle with making sense of situations" than to show them an algorithm, it has the advantage of helping students learn to think and reason about mathematical situations (p. 184). The methods they subsequently develop can be efficient, powerful, and generalisable. Many western education systems now explicitly discourage teachers from introducing algorithms before children have developed a deep and connected understanding of part–whole relationships within the number system (for example, Ministry of Education, 2007a; National Council of Teachers of Mathematics, 2000).

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Kieran's other four fraction sub-constructs include *ratio*, the idea of relative magnitude, necessary for understanding ideas about proportion and equivalence (as in renaming $\frac{3}{4}$ as $\frac{6}{8}$, $\frac{9}{12}$, $\frac{15}{20}$, and so on); *operator*, necessary for the multiplication of fractions (as in $\frac{3}{4}$ of 10 metres); *quotient*, necessary for problem solving (as in $\frac{1}{4}$ of 20 means the division of 20 by 4); and *measure*, necessary for addition of fractions (as in $\frac{3}{4}$ is the same as $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$). Another way of looking at fractions is to see them as both a *process* (for example, $\frac{3}{4}$ involves division of 3 by 4) and a *product* that results from a process ($\frac{3}{4}$ is the result of dividing 3 by 4) (see Verschaffel, Greer, & Torbeyns, 2006). The dual meaning of mathematical symbolism as *processes* to *do* and *concepts* to *know* has been captured in the term "procept" (Gray, Tall, & Pitta, 2000). Students can confuse fractions as *numbers* (that is, single quantities, as in $\frac{3}{4}$ of a litre) and fractions as *operations* (that is, proportions of quantities, as in $\frac{3}{4}$ of 10 metres). Physical materials can be used to model fraction tasks; different ways of modelling fractions include *region* or area models, *length* or measurement models, and *set* or collections models (van de Walle, 2004). Both region and length models involve *continuous* quantity, whereas set models involve *discrete* quantity. It has been suggested that one way to deal with all of this complexity is to define a fraction as simply "a point on a number line", allowing these different meanings of fractions to be deduced using logical reasoning (Wu, 2002). However, that has the disadvantage of excluding other powerful ways of coming to understand fractions using regions or sets.

Recent literature supports the idea that multiplicative thinking is essential for a deep and connected understanding of fractions, including proportions (Lamon, 2007; Thompson & Saldhana, 2003). It requires the recognition that "times" means "to envision something in a particular way – to think of copies (including parts of copies) of some amount" (Thompson & Saldhana, 2003, p. 104). It involves the realisation that there is an important reciprocal relationship, so that if quantity X is $\frac{1}{n}$ of a quantity Y, then Y is n times as large as X. Those who interpret fractions as "so many out of so many" (as in $\frac{1}{n}$ is "one out of n parts") are thinking of fractions *additively* instead of *multiplicatively* and will have difficulty dealing with situations where one quantity's size is a fraction of another quantity's size, when the quantities have nothing physically in common (for example, "the number of boys is what fraction of the number of girls?"). Hence, understanding *5 x 4* multiplicatively requires the understanding that the *4* in *5 x 4* is not just 4 ones (as in 20 = 4 + 9 + 7), but that the *4* is special because it is $\frac{1}{5}$ of the product (Thompson & Saldhana, 2003).

There is some debate in the literature about whether or not the teaching of algorithms is a good idea (Kamii & Dominick, 1998; Lappan & Bouck, 1998; Wu, 2002). Opponents of algorithms argue that some children never learn the algorithm and that those who can carry out algorithms don't always understand why or how they work, so they have little sense of when an algorithm is useful for solving a problem (Kamii & Dominick, 1998; Lappan & Bouck, 1998). They object to algorithms on the grounds that being told exactly how to do something "encourages children to give up their own thinking" (Wu, 1999, p. 4). Lappan and Bouck (1998) advocate the use of complex problems that encourage students to invent their own algorithms for adding and subtracting fractions. They argue that, although it takes more time to let students "wrestle with making sense of situations" than to show them an algorithm, it has the advantage of helping students learn to think and reason about mathematical situations (p. 184). The methods they subsequently develop can be efficient, powerful, and generalisable. Many western education systems now explicitly discourage teachers from introducing algorithms before children have developed a deep and connected understanding of part–whole relationships within the number system (for example, Ministry of Education, 2007a; National Council of Teachers of Mathematics, 2000).

For some teachers, providing instruction on algorithmic procedures may be the only method they have available to convey information about fractions to their students because their own subject-matter knowledge in mathematics is not sufficiently strong. There is now a growing body of literature that recognises teachers' own knowledge of mathematics as making an important contribution to their effectiveness as teachers (Ball, Hill, & Bass, 2005; Ball, Lubienski, & Mewborn, 2001; Goya, 2006; Moch, 2004; Shulman, 1986; Zevenbergen, 2005). However, it is not enough simply to be a good mathematician. Teachers also need to understand ways to support the learning of their students in mathematics; that is, they need to have strong pedagogical content knowledge (see Ball, 2006; Shulman, 1986). In a recent study of teachers' knowledge of fractions, Ward, Thomas, and Tagg (this volume) found that although two-thirds of the 44 teachers they surveyed successfully identified that $\frac{3}{5} + \frac{2}{3}$ does not equal $\frac{5}{8}$ (that is, they did not make the "add across" error identified by Smith, 2002), only four of the teachers (9%) were able to describe clearly the "key understanding" that a student making the "add across" error needs to acquire in order to add unlike fractions successfully. Five of the remaining 40 teachers (11%) gave a response that showed some understanding of fractions. However, the other 80% gave either an incorrect response (66%) or no response whatsoever (14%). The fact that one-third of the teachers accepted the "add across" error as an appropriate strategy to use for adding fractions should also be of concern. Even teachers working with the youngest primary-school students need to have a strong understanding of fractions if they are to help their students move towards a deep and flexible understanding of the number system. It is important to note that the teachers in the study by Ward, Thomas, & Tagg had previously participated in the professional development programme offered as part of the Numeracy Development Projects (NDP), an initiative designed to enhance teachers' subject-matter knowledge of mathematics as well as to provide them with tools to support the mathematics learning of their students.

It has been suggested that teachers tend to perpetuate the ineffective practices of their own teachers by instructing their students in mathematics the way that they themselves were taught at school (Grootenboer, 2001; Zevenbergen, 2005). Given the challenges of getting to grips with the multiple meanings of fractions, it is not surprising that fractions is an area that many teachers can find difficult to teach. This makes them particularly vulnerable to adopting the algorithmic approach to teaching mathematics that was typical of their own mathematics teachers.

A systematic examination of the kinds of errors that students make with fractions has been done to help teachers detect and correct common mistakes made by students working with fractions (see Brown & Quinn, 2006). For example, Brown and Quinn found that, of the 27 year 10 students who were unable to find a common denominator when adding unlike fractions, more than two-thirds of them added the numerators and then added the denominators (that is, the "add across" error identified by Smith, 2002). A fifth of them showed misconceptions related to equivalent fractions. Only about half of their year 10 cohort of 143 students was able to add $\frac{5}{12} + \frac{3}{8}$ successfully.

Data from the NDP show that generally, students' knowledge of fractions is limited, with less than a third of students (approximately 30%) at the end of year 8 able to recognise the equivalence of the fractions $\frac{2}{3}$ and $\frac{6}{9}$ while ordering a collection of mixed fractions (Young-Loveridge, 2005, 2006, this volume). Similarly, only about a third of students at the end of year 8 were able to work out that, if $\frac{2}{3}$ of a particular number is 12, then that number must be 18 (Young-Loveridge, 2005, 2006, this volume).

The study reported here is part of a larger project that set out to explore the perspectives of year 7–8 students in six intermediate schools (see Young-Loveridge, Taylor, & Hāwera, 2005; Young-Loveridge, Taylor, Sharma, & Hāwera, 2006). As part of the interview, students were given some mathematics tasks, including one that involved addition of unlike fractions ($\frac{3}{4} + \frac{7}{8}$). The purpose of the analysis presented here was to examine students' responses to the adding fractions task.

Method

Participants

This study focuses on the responses of 238 year 7–8 students in six urban intermediate schools in the North Island. The sampling technique was designed to ensure that there were at least as many Māori and Pasifika students as European. Table 1 shows the composition of the sample by school decile, gender, ethnicity, year level, and mathematics ability (as assessed by their teachers). Approximately one-quarter (24%) of the sample were European, one-third (33%) were Māori, just over one-third (37%) were Pasifika, and a tiny group had both Māori and Pasifika ancestry (3%) or were Indo-Fijian (3%). There were slightly more boys (54%) than girls (46%) and slightly more students from year 8 (55%) than year 7 (45%). Three of the schools had participated in the NDP and three had not (non-NDP). The deciles of NDP schools ranged from 1 to 4 while those of non-NDP were between 3 and 6. Students came from a range of mathematics ability levels, and assessment data from schools was used to categorise students as low, medium, or high (low: PAT stanine 1–3 or level 3P and below on AsTTle; medium: PAT stanine 4–6 or level 3A to 4P on AsTTle; high: PAT stanine 7–9 or level 4A and above on AsTTle).

Table 1
Composition of the Sample by School Decile, Gender, Ethnicity, Year Level, and Mathematics Ability

School	**Gate***	**Hill***	**Ivy**	**Jute**	**Kite**	**Lake***	**Overall**
Decile	3	4	6	4	3	1	
Total	59	39	57	47	19	17	**238**
Gender							
Girls	26	20	24	22	10	7	**109**
Boys	33	19	33	25	9	10	**129**
Ethnicity							
European	15	17	12	13			**57**
Māori	21	11	33	14			**79**
Pasifika	22	8	9	13	19	17	**89**
Māori/Pasifika	1	3	3				**7**
Indo-Fijian				7			**7**
Year level							
Yr 7	28	21	20	23	7	9	**108**
Yr 8	31	18	37	24	12	8	**130**
Maths ability (as assessed by their teachers)							
High	7	15	7	7	8	0	**44**
Medium	37	13	30	26	7	8	**121**
Low	15	11	19	14	2	9	**70**
Unknown			1		2		**3**

*Schools that had participated in the NDP

Procedure

Schools were asked to nominate students from across a range of mathematics levels within each of the three main ethnic groups. Students were interviewed individually for about 30 minutes in a quiet place away from the classroom. They were told that the interviewer was interested in finding out their thoughts about learning mathematics. As well as questions about the students' views, a word problem involving the addition of $\frac{3}{4}$ and $\frac{7}{8}$ was given, as follows:

> Sione and Tama buy two pizzas. Sione eats $\frac{3}{4}$ of a pizza while Tama eats $\frac{7}{8}$. How much pizza do they eat altogether?

The task was read to students and they were offered a pencil to write down their problem-solving processes. They were then asked to explain to the interviewer their solution strategy, and these conversations were recorded on audiotape. Interviews were transcribed and the transcripts subjected to a content analysis to identify common themes coming through in the students' responses.

Results

Students' responses to the questions were organised according to common patterns emerging from the data. The code at the end of each excerpt indicates the school's name (initial letter) and the student's individual number, as well as year level and gender. Table 2 shows the number of students who responded in particular ways to the task.

Students' Strategies for Adding $\frac{3}{4}$ and $\frac{7}{8}$

Correct answer (13.4%)

A variety of answers were judged to be correct, including $1\frac{5}{8}$, $\frac{13}{8}$, $6\frac{1}{2}$ quarters, one and 2.5 quarters, and 1.6. (Note: Although 1.6 is only an approximation to the correct answer, it was accepted as correct. Likewise, $6\frac{1}{2}$ quarters, and one and 2.5 quarters were accepted, even though they violate the principle that the numerator and denominator should be whole numbers, because they are alternative expressions for the ratios $\frac{13}{8}$ and $1\frac{5}{8}$.) Only 32 students gave a "correct" answer to the problem. Several different approaches were taken to find the correct answer. Some students chose to use the "make a whole" strategy, similar to the "make ten" strategy, where part of one pizza was joined with the other pizza to make it into a whole pizza and the remaining fractional part calculated. Others chose to find a common denominator before adding the two fractional parts. This group was subdivided into two sub-groups: those who seemed to have a strong conceptual understanding of fractions and used appropriate fraction language to describe their strategy, and those who used language indicative of a procedural approach to solving the problem. A small group of students used quarters as the common denominator for adding the two fractional parts together.

Used the "make a whole" strategy and then calculated the leftover fractional part

Five students used the "make a whole" strategy successfully, partitioning one of the fractions so that part of it could be put with the other fraction to make it into a whole and the remaining fractional part calculated.

> Six plus seven because you take one off the six, which will make that [the six] five, plus that seven [from the $\frac{7}{8}$ pizza], which is eight over eight which is one [whole], and then the rest of it will be five over eight. (G33, yr 8 girl)
>
> [Drew two pizzas] Sione eats three-quarters, and I think that means eighths. Two of those eighths are left. One, two, three, four, five, six. Oh, here there's ... one, two, three, four, one and five-eighths [counting the remaining eighths in Tama's pizza]. (G46, yr 8 girl)
>
> Well, I took one of these and put them in that [$\frac{7}{8}$ pizza] and then that's one [whole], and then that would be five of them. (I14, yr 8 boy)
>
> [Drew two pizzas] I just shaded in seven-eighths and three-quarters and then I took one-quarter away from here [$\frac{7}{8}$ pizza], put it there [with the $\frac{7}{8}$ pizza] and it's one, and then I just added up five there, so I put five on. (I16, yr 8 girl)

Table 2
Number of Students Who Responded in Particular Ways to the Adding Fractions Task

School	**Gate***	**Hill***	**Ivy**	**Jute**	**Kite**	**Lake***	**Overall**	**%**
Total	*59*	*39*	*57*	*47*	*19*	*17*	**238**	
Correct answer								
Used "make a whole"	2		3				**5**	2.1
Used eighths as common denom. with understanding	1	4	1	1	2		**9**	3.8
Used eighths as common denominator with procedural explanation	1	2	3	1	1		**8**	3.4
Used quarters as common denominator			1	3	1		**5**	2.1
Other				2	1		**3**	1.3
No explanation	1			1			**8**	0.8
Total correct	*5*	*6*	*8*	*8*	*5*	*0*	*32*	*13.4*
% Correct	*8.5*	*15.4*	*14.0*	*17.0*	*26.3*	*0.0*	*13.4*	
Incorrect answer								
Used fraction equivalence	4	3	4	5	1		**17**	7.1
Ten-whole confusion	4			1		1	**6**	2.5
Estimated or guessed		2		1	2		**5**	2.1
Used "make a whole" strategy	1	1			1		**3**	1.3
Made a procedural error			1			1	**2**	0.8
Added nums/denominators ("add across" error)	13	7	12	15	4	3	**54**	22.7
Miscellaneous	9	5	11	6	1		**32**	13.4
Total incorrect	*31*	*18*	*28*	*28*	*9*	*5*	*119*	*50.0*
% incorrect	*52.5*	*46.2*	*49.1*	*59.6*	*47.4*	*29.4*	*50.0*	
No attempt	**23**	**15**	**21**	**11**	**5**	**12**	**87**	**36.6**
% no attempt	*39.0*	*38.5*	*36.8*	*23.4*	*26.3*	*70.6*	*36.6*	
Mathematics ability								
% high mathematics score	11.9	38.5	12.3	14.9	42.1	0.0	**18.4**	
% medium mathematics score	62.7	35.9	52.6	55.3	36.8	47.1	**45.2**	
% low mathematics score	25.4	28.2	33.3	29.8	10.5	52.9	**35.1**	

*Schools that had participated in the NDP

Used eighths as a common denominator and had strong conceptual understanding

Although a total of 17 students renamed $\frac{3}{4}$ as $\frac{6}{8}$ and used eighths as a common denominator for the addition of $\frac{6}{8} + \frac{7}{8}$, the explanations of the students differed markedly. Half of the students (n = 8) used language that suggested they had a strong conceptual understanding of adding fractions (see Figure 1 for the written explanation of one of the students in this group: K04). They referred frequently to the name of the fractional part as "eighths" and rarely or never used language such as "out of" or "over" when referring to the symbolic representation of fractions.

> I'm just working out how much that would be in eighths. So that would be thirteen-eighths. (H28, yr 8 girl)
>
> We go six-eighths and seven-eighths so they eat ... thirteen-eighths, then you could change that to one and five-eighths. (H29, yr 8 boy)
>
> I would make that into eighths to make it easier so it would be six-eighths, and I'd go six plus seven which is thirteen-eighths, which means a whole and five pieces. And then if there's eight pieces, I just minus the five off the second pizza because they've eaten an extra five of the second pizza, so you go eight minus five equals three. [Appeared to be working out how much pizza was not eaten, so was asked about how much was eaten] A whole and five pieces, eighths. (H30, yr 8 boy)
>
> I doubled the four to make it into an eight, and then I doubled the three to make it six-eighths, and then I added the two together. That gave me thirteen-eighths. And then I change it into a, I forget what it's called, this bit ... One and five-eighths. (K04, yr 8 boy)

> Thirteen-eighths. Because eight is two times four, I just doubled it so then that's six and that, that's eight, and then I added six to seven, which is 13 and then you put it over eight. [When asked about another way to work it out, gave the answer as 6.5 over 4] (G36, yr 8 girl)

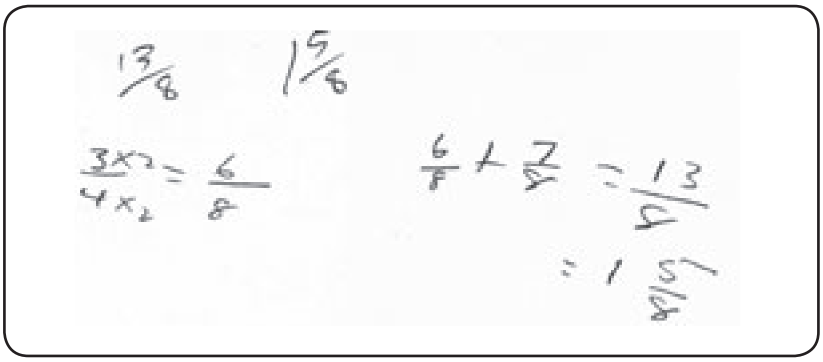

Figure 1: An example of the written explanation of a student who appeared to have strong conceptual understanding (K04)

Used eighths as a common denominator and gave a procedural explanation

Eight of the 17 students who renamed $\frac{3}{4}$ as $\frac{6}{8}$ so that they could add it to $\frac{7}{8}$ used language in their explanations that suggested the use of a highly procedural approach to solving the problem. For example, students in this sub-group described the use of "times by two" in converting quarters to eighths or "minus eight" from 13 to work out how many fractional pieces were left after the improper fraction was converted to a mixed number consisting of one whole and a fractional part. Virtually all of them used "over" to refer to the symbolic representation of fractions, as in "five over eight", rather than referring to the number and name of the fractional parts, as in "five-eighths". One student (H32; see Figure 2) multiplied the two denominators 4 and 8 to find a common denominator of 32 rather than using the relationship of eighths to quarters. One boy (K06) seemed to believe that the numerator must always be smaller than the denominator, as was evident from his comment after working out an answer of thirteen-eighths, that "you can't do that, so you ... minus it from eight." It is possible that some of these students did not fully understand the reason for using a common denominator or the acceptability of both an improper fraction and a mixed number as ways of expressing fractional number.

> I do times that by two to get eight, so I timesed that by two, and then I just added six and seven together which gave me 13, and then that was, I've forgotten what it was called ... [Interviewer: Improper?] Yeah, an improper fraction and then I just went 13 minus eight equals five, so I went one and five over eight. (H24, yr 8 girl)

> [On paper, wrote 32 as the denominator after multiplying four times eight, then wrote 24 for one numerator after multiplying eight times three, and 28 for the other numerator after multiplying four sevens, added them to get 52 in total, and wrote $\frac{52}{32}$] There is 20 left over, 20 over 32, and then I just broke it down to get five-eighths [he halved $\frac{20}{32}$ to get $\frac{10}{16}$, then halved again to get $\frac{5}{8}$], one and five-eighths. (H32, yr 8 boy)

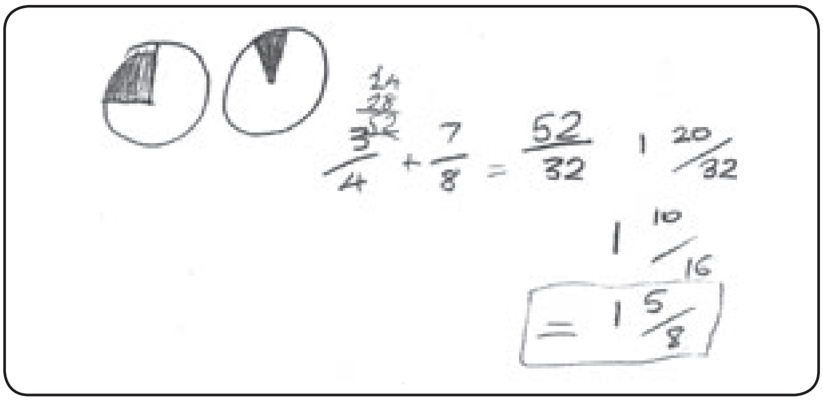

Figure 2: An example of a correct response showing the use of an algorithm (H32)

> I would do the numerals for both of them the same first. That's three over four. I would times it by two to get six over eight and then I would plus seven over eight plus six over eight which gives me 13 over eight and I would break it down to five over eight. It's one and five over eight. (J03, yr 8 girl)

> They eat ... Sione eats most of it except for one-eighth, oh no, he eats three-quarters, so there's one-quarter and one-eighth left. And altogether they would eat one whole pizza and five-eighths I think. Because there's one. I found out how much was left then I divided a pizza into eighths because I know what quarters are, and 'cause that's timesed by two. Like the bottom one, that's just double the amount to that so it's the double amount of pieces so you just cut each quarter into half. (I28, yr 8 girl)

> One and five-eighths. I changed three-quarters with six-eighths. And then I plus six-eighths with seven-eighths. And then I added it all together and it came out with fifteen-eighths, but then you can't do that. No, not 15. Thirteen-eighths and you can't do that so, you take out, you minus it from eight. And so, it becomes a five and put a one in front of the five so, it'll be five-eighths. One and five-eighths. (K06, yr 8 boy)

Used quarters as a common denominator

Five students chose to use quarters as the common denominator, changing seven-eighths into three and a half quarters in order to add it to three-quarters, and getting a total of six and a half quarters. Two of these students then worked out the answer as a mixed number, one and 2.5 (J16) or one and two and a half (J17). A sixth student, whose initial strategy involved using eighths, when asked for a different way of solving the problem, gave an answer of "6.5 over 4" (see G36 above). Four of the six students used language such as "out of" or "over" when describing their strategy.

> Six and a half quarters. I halved seven-eighths and that was three and a half quarters and then added three and a half to three. (J14, yr 7 boy)

> I know there's like half left over, sort of like six and a half quarters. I know that three, four, and then over here two-eighths equals a quarter so then I just do it like that. I just split it into halves. [Asked what the final answer was] Six and a half quarters. [Asked about other ways of saying it] Yes there is another way but I just need to think. Thirteen-eighths. I just timesed the three by two and then added it to the seven. (I21, yr 8 boy)

> They'd eat six point five. [Interviewer: six point five?] Yeah, six and a half [Interviewer: six and a half?] slices. [Interviewer asks for further explanation.] I halved it. I halved all this. [Interviewer: half of seven-eighths is?] It would be three point five over four. One pizza and two point five [Interviewer: two point five?] out of four. (J16, yr 8 boy)

> [Wrote 1 $2\frac{1}{2}$] You just halve those. Half of seven is three and a half over four, and then you added that to the three-quarters. And because six is more than the four, you need to, that means it's a whole plus ... one whole [pizza] and two and a half. (J17, yr 8 girl).

> There's a quarter pizza left on that one and on this one there's one-eighth left and then. Two, there'd probably be four-eighths left and then two out of eight and that's three-eighths. There'll be three out of eight left and three out of eight would ... [Asked to explain] He ate three and a half, six and a half, I think. I think it might be six and a half, I have no idea. I think it might be six and a half. (K01, yr 8 girl)

Other correct answers

There were also some idiosyncratic strategies used. For example, one student (J47) responded using decimal fractions, initially thinking there would be 1.25 pizzas but eventually settling on an estimate of 1.6 pizzas. One student (K02) started by making the "add across" error, adding the numerators 6 and 7, and putting the sum (13) over the sum of the denominators (16). However, during the course of her explanation, she self-corrected to give an answer of $\frac{13}{8}$, then converted this to a mixed number. Initially, she did not refer to the size of the pieces ("one full one and five pieces"), but in her written explanation it is clear that she was working with eighths (see Figure 3). Interestingly, her comment that "13 over eight doesn't work", before converting $\frac{13}{8}$ to a mixed number, suggests that she does not understand that improper fractions are acceptable ways of expressing fractional numbers.

> I think, like 1.25 pizzas, I think. Because the eight is the whole pizza so then there's, because that one is three-fourths and double that would be sort of expanding it so that's eight over six or six over eight and then six plus seven is 15, no, 13. And it would be 13 minus eight is five so then there's five over eight left. It's not 1.25, oh dammit, I forgot that would be 1.5 or 6, I think, like 1.6 pizzas, or something round there. [Asked to explain further] So it's an improper fraction. So there's more than one so you have to take away the eight from the 13. It's five. That's five-eighths of the pizza. (J47, yr 8 boy).

> Thirteen over 16, I think. I'm not too sure. [Asked to explain] I doubled this so it was the same as ... one pizza and five pieces. [Asked to explain further] I doubled that fraction, like I did here. And then I added the six and the seven, which gave me 13. And 13 over eight doesn't work, so I figured out the difference between 13 and eight and that gave me five, and then I took the five away from the 13, which gave me eight over eight, which is one full one and five pieces. (K02, yr 8 girl)

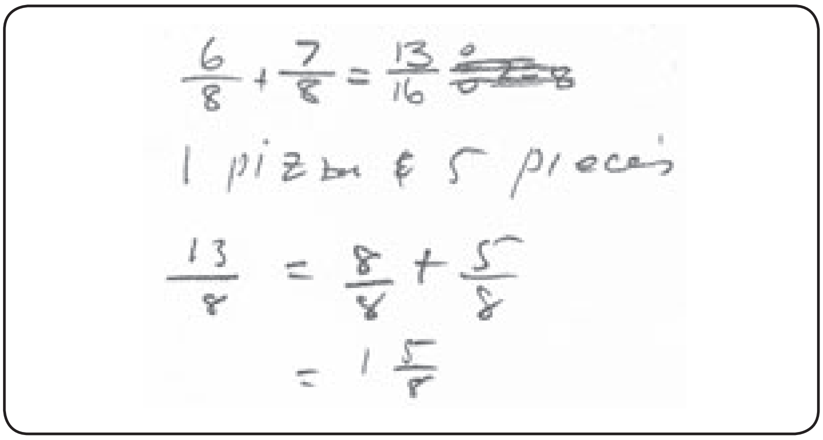

Figure 3: An example of a student who self-corrected her response during her explanation of her solution strategy (K02)

Correct answer without an explanation of the strategy

Two students were not able (or willing) to explain the strategy they had used to solve the problem. One boy (J26) claimed that he got his answer through making a lucky guess, but this response may have been because he was not completely confident he had solved the problem correctly.

> I don't know how. (G05, yr 8 girl)

> It's just a guess, one whole and five-eighths, I think. I'm not too good at fractions so I just thought of that. It was a lucky guess. (J26, yr 7 boy)

Incorrect answer (50.0%)

More than half of the students (n = 119) were unsuccessful in their attempt to add the two fractions. Several distinct strategies were evident from the students' responses. Some of the students who did not find a correct answer nevertheless showed an awareness of equivalent fractions. The majority of incorrect responders made the "add across" error, adding numerators and/ or denominators. A small group appeared to confuse the "make a whole" strategy with the "make ten" strategy (probably a familiar strategy used for adding whole numbers). Some students chose not to work out a precise answer, preferring instead to estimate (or guess) a close approximation to the answer. Three students attempted to use the "make a whole" strategy but miscalculated the remaining fractional part. Two made a procedural error while calculating the sum of the two fractions. Thirty-two students gave an idiosyncratic response that was unlike any other response given by someone in this group.

Used fraction equivalence to convert three-quarters to six-eighths

Seventeen students initially renamed $\frac{3}{4}$ as $\frac{6}{8}$, showing their awareness of fraction equivalence. However, they then went on to make an error, adding $\frac{6}{8}$ and $\frac{7}{8}$ together. Typically, they added the numerators and then added the denominators (the "add across" error) to get an answer of thirteen-sixteenths. One student (I25) began with an answer of "thirteen-eighths" but, after explaining his strategy, wrote "$\frac{13}{16}$". Some students ignored the denominator, adding just the two numerators (for example, H08). The language used by these students reflects a procedural approach with an emphasis on rules. For example, one student (G42) stated that "you can't have halves inside a fraction," whereas another (G44) stated that "you have to make the denominator the same."

> He ate three-quarters and that was six and he ate seven-eighths, so six plus seven is 13, and eight plus eight is 16 [wrote $\frac{13}{16}$]. (J34, yr 8 boy)

> Six and a half eighths, and then you have to change that again by doubling it so it's 16, that's thirteen-sixteenths. [Asked to explain further] Because you can't have halves inside a fraction. (G42, yr 8 boy)

> To me, it actually depends on whether there's four pieces on the first pizza or eight pieces on the second, so it would be, and that would be ten, it would be about, if that's eight pieces, then there must be eight pieces on that one as well. So that would be six and seven, and then I would add the seven and six together and get 13 out of 16. (H11, yr 8 girl)

> Thirteen-eighths, 'cause you know how to take in quarters. I cut them into eighths, this halves each part of it, and then double that number so I double that one. Doubled the six [? meaning doubled the three to get six]. Oh wait, I got that one wrong. That's an improper fraction isn't it? Oh the eight, thirteen-sixteenths. [Starts again] I cut each quarter in half ... I just plussed the two eights together and I plussed the six and seven to get 13 ... I was counting the two pizzas together, I just wanted to split them up ... I want to change that eight to 16 [wrote $\frac{13}{16}$] because I plussed these two together. (I25, yr 8 boy)

> They eat three-eighths altogether. [Asked to explain] Oh no, they <u>don't</u> eat three-eighths. They leave three-eighths, so they eat five-eighths altogether. [Asked to explain further] I doubled three-quarters to six-eighths. And I just saw that six-eighths, it's two-eighths away from a whole, and seven-eighths is, oh is seven over eight, is one over eight, yes, is a whole. So I added one over eight and two over eight which is three over eight, and then three over eight minus ... eight over eight is five over eight. [Asked to explain further] Three-quarters, so he eats this much and seven-eighths, half of that is 3.5 fourths, which would mean he'd eat that much and one of these so, that still there and that would be that one, so there'd be .5 of a pizza left, no um, one, no there'd be a quarter and a half which is three. There'd be eighths left, so there'd be three-eighths of one pizza left. [Asked how much pizza was eaten] Um, four, six, six and a half eighths, which is, yes, six and a half eighths which is thirteen-sixteenths. So they ate thirteen-sixteenths of a pizza ... So you can just double that so that's eight. Oh six-eighths and you just go six-eighths plus seven-eighths is thirteen-sixteenths. (G43, yr 8 boy)

> I just went, because you have to make the denominator the same, put four up to eight and then you just, because that's doubling it, so double three which is six-eighths and then add seven-eighths and six-eighths together, which is thirteen-sixteenths. (G44, yr 8 girl)

Ten–Whole confusion

Six students gave answers that revealed some confusion between the number of fractional parts making up the whole and the base-ten nature of the number system. This may have been the result of trying to use the "make a whole" strategy but confusing it with the "make ten" strategy they had previously used for adding whole numbers. For example, several students commented on particular number combinations that make ten (such as 7 and 3) instead of referring to the number of pieces needed to make a whole when the pieces are eighths. It may be for this reason that two other students (from the "miscellaneous" category) answered that adding $\frac{3}{4}$ and $\frac{7}{8}$ made "a whole one" (G51 and G57).

A whole and two pieces. [? Treating the three-quarters as three-eighths, converting the $\frac{7}{8}$ pizza to a whole using one of the quarter pieces, leaving two quarter pieces left over?] [Asked to explain] Seven pieces plus three pieces equals one piece [? Thinking of a whole pizza as being like a "tidy ten", made up of combinations such as 7 and 3?] and then you get two more. [Asked to explain further] Three and four is seven, and seven and eight is 15 [? Adding the numerator and denominator for each of the two fractions?] (G29, yr 7 boy)

I got one and three-quarters or one and a half. I'm not really sure. [Asked if he was making an estimate] Because what I got was two pizzas and a half, which can't be right. [Asked to explain how he got this] I just added those two – seven and three so that makes ... seven and three which makes a whole, and then I went eight plus four, which is 12, which is another whole and a bit. (G32, yr 7 boy)

Another student (H12) showed a similar confusion in reading a "teen" number as one whole and some fractional pieces (the number corresponding to the single-digit quantity beside the "1"), implying that 13 means one whole and three fractional parts.

Ten [Asked ten what?] pieces. [Asked to explain] Oh, 13. [Asked to explain further] Because they just left eight pieces on a pizza. That's half there. [Interviewer says "and this is three-quarters."] That's the same as six eighths and seven eighths, so that equals 13. I plussed those two numbers. Oh, they ate one whole pizza and three-eighths. It's a mixed number. [Asked "What's a mixed number?"] One whole number and ... [Interviewer: So is this the <u>one</u> here, you're saying that <u>one</u> comes from the 13 there?] Yeah. So that's an improper fraction I think. [Interviewer: So 13 is equal to one and three-eighths?] Yes [had written $\frac{6}{8} + \frac{7}{8} = 13 = 1\frac{3}{8}$]. (H12, yr 7 girl)

Estimated or guessed the answer

Five students came up with answers that suggested they had tried to guess the answer rather than attempting to calculate a precise answer. This strategy may have been chosen because of uncertainty about an appropriate way of calculating the answer.

Is that a third of the pizza left? That's three-quarters of a pizza and there's a quarter of it left, and so then there's another piece left that's about three-quarters, that's about a quarter. A third of it left. [Asked how much pizza they ate altogether] They had about a pizza and two-thirds. (I41, yr 7 girl)

About a pizza and a quarter. Something like that. (I50, yr 7 boy).

About a whole. [Was asked to explain] More than a whole. [Was asked to explain further] I forgot. [Interviewer offered him paper to write on.] Well, that there's three-quarters and that's seven-eighths so there'd be a little bit left in that one. It's about one and a half. (H33, yr 7 boy)

One and seven-eighths. [Asked to explain] I don't know. Oh one and six-eighths. (K05, yr 8 girl)

One and a quarter, I think. I don't really know. I just took away, I guess. (K09, yr 8 girl)

Tried to "make a whole" but miscalculated

Three students produced an incorrect answer as a result of calculation errors made while trying to use the "make a whole" strategy.

I looked at three-quarters, and two-eighths equals a quarter, and take two away from eight [instead of seven] you get, six eighths. One whole and six-eighths. (K17, yr 7 boy)

[Drew a diagram of two pizzas] That's one pizza and seven-eighths, that goes one ... one, two, three, four, five, six, seven, eight, and then shade seven. Seven and that one, shade. Shade that one in, and then that one. That one, if we just put that into there, that's one whole and a half. One whole pizza and half a pizza. (G09, yr 8 girl)

One and a quarter. Because there's eight pieces in a pizza and Tama ate seven of them, and there's one more piece on that pizza, and Sione ate the last one from that eight and then two from the eight on the second pizza. The two they ate from the second pizza was a quarter. (H16, yr 8 boy)

Made a procedural error while trying to use an algorithm

Two students made errors while trying to use an algorithm, apparently not recognising the relationship between quarters and eighths. Lack of basic-facts knowledge (of 8 x 4) meant that the algorithmic procedure resulted in an incorrect answer (L02 thought 8 x 4 was 36; see Figure 4).

> [Wrote $(\frac{3}{4} \times \frac{8}{8}) + (\frac{7}{8} \times \frac{4}{4}) = \frac{24}{36} + \frac{28}{36} = \frac{52}{36}$. Wrote 52 – 36 = 16 using vertical written algorithm. Finally wrote $1\frac{16}{36}$] (L02, yr 8 boy)

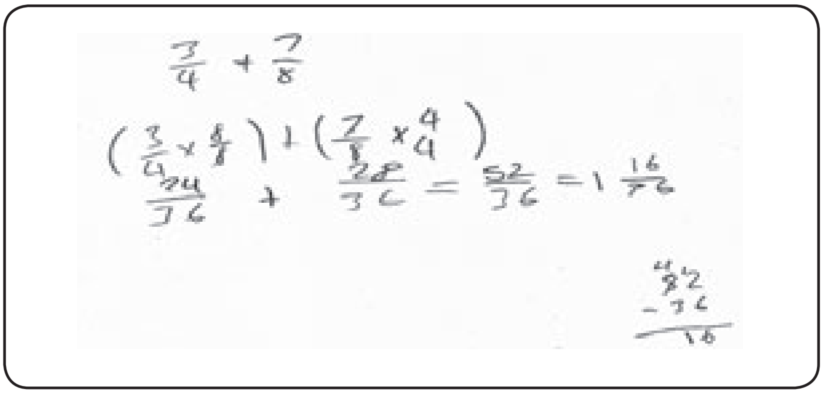

Figure 4: An example of an incorrect response showing the use of an algorithm (L02)

Added numerators and /or denominators

Adding the numerators and/or the denominators was the most frequent strategy used to get an incorrect answer, with at least 54 students using some form of this particular strategy (some of those who used fraction equivalence also went on to make this "add across" error). The most popular version of this strategy was to add three and seven for the numerator and four and eight for the denominator, giving an answer of $\frac{10}{12}$ (see Figure 5). Other variations on this strategy produced responses such as $\frac{10}{16}$, $\frac{12}{10}$, $\frac{10}{8}$, $\frac{20}{24}$, and 22 (the sum of all numerators and denominators). It was interesting to observe that five of the students who responded with $\frac{10}{12}$ then simplified it to $\frac{5}{6}$.

> [Wrote 3 + 7 = 10, 8 + 4 = 16, $\frac{10}{16}$] (G38, yr 7 girl)
>
> [Wrote 3 + 7 = 10, 4 + 8 = 12] The bottom number has to be bigger. (G40, yr 7 boy)
>
> [Wrote 3 + 7 = 10, 4 + 8 = 12, $\frac{10}{12} = \frac{5}{6}$] (H37, yr 8 boy)
>
> [Wrote $\frac{3+7}{4+8} = \frac{10}{12}$, $\frac{5}{6}$] (H40, yr 8 girl)
>
> [Drew two pizzas and shaded the eaten part. Wrote $\frac{3}{4} + \frac{7}{8} = \frac{10}{12}$] (J36, yr 7 girl)

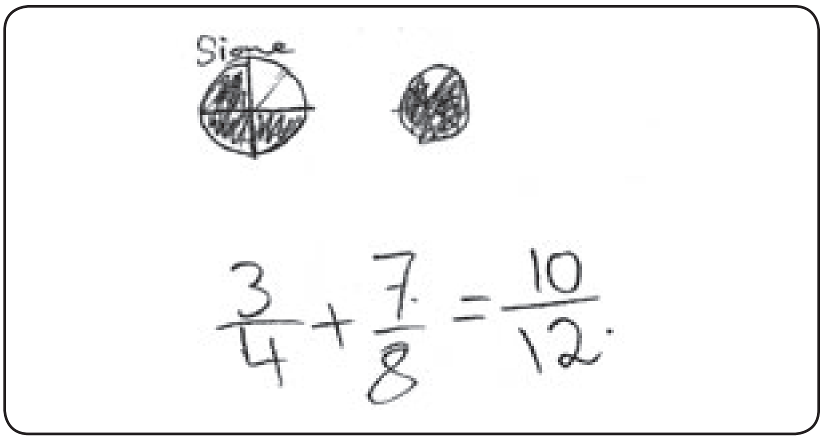

Figure 5: An example of a response showing the "add across" error (J36)

Miscellaneous responses

This category included unusual responses that were difficult to interpret, such as $\frac{1}{2}$, $\frac{45}{48}$, 2 pizzas, $\frac{1}{2}$ and $\frac{3}{4}$, 2 and a half, $\frac{6}{4}$ of 2 pizzas, $\frac{3}{4}$ and $\frac{1}{8}$, ten and a half, $\frac{9}{10}$, just over $\frac{3}{4}$, $\frac{3}{4}$ of 2 pizzas, and $\frac{52}{64}$ (the sum of cross multiplying with a denominator of eight times eight).

> Sione ate three-quarters and Tama ate seven-eighths, so that would be one whole pizza altogether. [Asked to explain] Well, seven-eighths is the same as three-quarters, well I think it is. So it would be one and a half. Yeah one pizza and a half. [Asked where the half came from] Like a decimal. I just did three times five, 15, and that would be one point five and that would be one and a half. (H05, yr 7 boy)

> I'm really bad at fractions. I've probably got it wrong. You have the pizza and you divide it into four because Sione ate three-quarters so she eats that part, that part and that part, and then you have Tama and he eats seven-eighths of the pizza so he eats that part and there's only that bit and that bit left. [Asked how much did they eat altogether] It's nearly two but not quite, I'm not sure 'cause I'm really bad at fractions ... I usually try really hard to understand what's going on but the fractions and stuff I don't get it and as much as I go over it and stuff, I just don't get it. [Interviewer comments on the usefulness of drawing pictures.] I'm pretty good with visualising things, like if sometimes we have problems, they'll give you like a set or something and they'll say what shape does it make and I can usually see in my head what shape it's going to make. (H31, yr 7 girl)

No attempt (36.6%)

More than a third of the sample (87 students) chose not to respond to the pizza problem. This group included three students who tried to draw the pizzas but then responded "Don't know" (see Figure 6).

Figure 6: An example of a drawing made by one of the students who eventually responded "Don't know" (J08)

As part of the ethics process, the students had been told that they could skip any question they did not want to answer. We respected that decision and did not press the students to make responses to the mathematics tasks. On reflection, we think it would have been useful to ask the students to draw something to show each of the two fractional quantities, even if they were not able to add them together. It is possible that this might have revealed some understanding of fractions by the students who chose to not even attempt the task. To find out which of the "no attempt" students might have been able to do the fractions task successfully if they had chosen to, we examined the relationship between students' responses to the fractions task and their assessed mathematics ability.

Relationship of Responses to Fractions Task with Assessed Mathematics Ability

Analysis of the mathematics assessment information showed that approximately ten (11.5%) of the students who made no attempt to do the task might have succeeded on the fractions task if they had tried to do it. These ten students had been assessed by their teachers as being at stage 7 or higher on the Number Framework, or in the upper third of the distribution (see Table 3). On the other hand, almost a fifth (19.2%) of the group who did not get a correct answer were from this high-mathematics-achievement group. It is interesting to note that a quarter (25.8%) of the students who found a correct answer were not among the high mathematics achievers.

It is clear from Table 3 that the majority of students who responded with a correct answer were among the highest mathematics achievers for their year level, with almost three-quarters (74.2%) of correct responders having been assessed as high in mathematics by their teachers.

Table 3
Numbers of Students Who Were High Mathematics Achievers for Each Response Type

Type and level of mathematics assessment	Correct response	**Response type** Incorrect response	No attempt
Number of students in group	n = 32	n = 119	n = 87
NDP Framework			
Stage 7	3	7	6
Stage 8	5	6	2
Total stage 7+	*8*	*13*	*8*
AsTTle tool			
Level 4A	2	3	1
Level 5B	1	1	
Level 5P	2	1	
Level 5A	1		
Level 6B		1	
Total level 4A+	*6*	*6*	*1*
Progressive Achievement Test			
Stanine 7	4	3	1
Stanine 8	3	1	
Stanine 9	2		
Total stanine 7+	*9*	*4*	*1*
Number of high maths achievers	*23*	*23*	*10*
Percentage of high maths achievers	*74.2%*	*19.2%*	*11.5%*

Discussion

Overall, relatively few students appeared to have a deep understanding of fractions or fraction computation. This finding is consistent with those writers who argue that fractions present a major challenge to students and to their teachers (Davis et al., 1993; Hunting, 1994; Lamon, 2007; van de Walle, 2004). However, this finding has some important implications for the implementation of New Zealand's new draft curriculum document, where the expectation is that students at level four should be able to solve problems using multiplicative and simple proportional strategies (Ministry of Education, 2006). Fewer than a third of year 8 students are at stage 7, advanced multiplicative, (see Young-Loveridge, 2005, 2006, this volume) and therefore few students in this study were able to add $\frac{3}{4}$ and $\frac{7}{8}$ fluently. The expectation that students at level four should be multiplicative thinkers is based on research evidence showing that students cannot engage with algebra effectively if they are not multiplicative thinkers (for example, Lamon, 2007; Wu, 2002). Hence, there is clearly a need to provide additional professional development for teachers working at the upper primary and intermediate levels (years 5–8) to help them to appreciate the importance of multiplicative thinking and provide them with instructional support in this area. The revisions to Book 1 (The Number Framework) and Book 6 (Teaching Multiplication and Division) are designed to do just that (see Ministry of Education, 2007b, 2007c). The Ministry of Education's fee-subsidy scheme, which provides some financial support to offset the costs of teachers doing further university study in mathematics education, may also help, but it needs far more publicity as well as support from schools if it is to have an appreciable impact on teachers' understanding of the upper stages of the Framework. Lamon (2007, p. 633) comments that "we educators are failing miserably at teaching the most elementary multiplicative concepts and operations."

In her longitudinal study using a design experiment, Lamon (2007) found that in the first two years of the programme, which was designed to build students' understanding of the central multiplicative structures involved in fractions, the students were outperformed on fraction computation by rote learners taught using a traditional approach that included rules and algorithms. However, in the longer term, the students whose instruction was focused on building meaning and sense making surpassed the rote learners. In the light of these findings, we need to exercise caution over how much improvement it is reasonable to expect from the NDP professional development programme in relation to the multiplicative and proportional domains of the Framework.

The findings of relatively limited fraction understanding by students in the present study raise some important questions about teachers' subject-matter knowledge in the domain of fractional number. Lamon (2007, p. 633) points out that many adults, including teachers, "struggle with the same concepts and hold the same primitive ideas and misconceptions as students do." This was borne out by a recent study of teacher knowledge about fractions (Ward, Thomas, & Tagg, this volume) where it was found that the majority of teachers in the study (91%), all of whom had participated in the NDP professional development programme several years prior to the study, were unable to articulate the key understanding needed to help students add fractions with unlike denominators. These two studies taken together underline the importance of strengthening teachers' subject-matter knowledge of fractions in particular. It is vital that this be made an urgent priority within pre-service teacher-education programmes as well as within in-service programmes.

A notable strategy used by a small group of students in the present study to add $\frac{3}{4}$ and $\frac{7}{8}$ was the "make a whole" strategy, whereby part of one fraction was put with the other fraction to make a whole and the remaining fractional parts counted (Huinker, 1998). This strategy is similar to the "make ten" strategy used in whole-number computation, where one of the addends is partitioned so that one of its parts can be joined with the other addend to make ten (or a multiple of ten), as in $9 + 5 = 9 + 1 + 4 = 10 + 4$ (see Thompson, 1999, 2000).

One group of students in the present study who found the correct answer did so using what appeared to be procedural knowledge rather than conceptual understanding. A crucial means of deciding whether or not the student's explanation was procedural was if their language suggested the application of taught procedures (Smith, 2002). Even though many of these students found the correct answer, their responses seemed very mechanistic and rule-based rather than being fluent and grounded in a deep and connected conceptual understanding of fractions. The absence of reference to the names of the fractional parts (for example, quarters and eighths), instead using positional language to describe the written symbols produced by the fraction computation (for example, "13 over eight", "five over eight"), was taken as an indication that they had used an algorithm involving the manipulation of the digits within the fractions according to a set of rules rather than carrying out meaningful computation with fractional quantities. This is consistent with Lamon's (2007) comment that research with students who have had at least five years of traditional instruction in mathematics shows that reasoning strategies tend to be replaced by rules and algorithms by the time students have been at school this long. Mack (1990) also found that her students referred to fractions in terms of the number of pieces rather than commenting on the size of the pieces.

A small group of students in the present study confused the "make ten" strategy with the "make a whole" strategy. These students seemed to have confused the decade-based (place value) structure of the number system with the particular kind of part–whole relationships found between the fractional parts and the whole when the whole is partitioned into parts other than ten (for example, eighths). Several students seemed to think that seven plus three made a whole, ignoring the denominators altogether. Another student thought that the number 13 meant that there was one whole and three fractional parts. This particular misconception suggests that a strong emphasis on the decade-based structure of the number system could be at the expense of other part–whole relationships that students need to understand. Teachers need to be aware that this potential misconception is one that some of their students may develop.

For a sizeable group of the students who gave an incorrect answer to the fractions tasks, it was clear from their explanations that they were aware of the need to find equivalent fractions when adding fractions with unlike denominators. Most of them knew that they needed to convert $\frac{3}{4}$ to $\frac{6}{8}$ in order to add it to $\frac{7}{8}$ (Huinker, 1998). A smaller group were aware of the equivalence of $\frac{1}{4}$ and $\frac{2}{8}$. The idea of fraction equivalence is a key component of the part–whole sub-construct for fractions (Behr et al., 1983).

About half of the students gave an incorrect answer to the fractions task, and half of these (about a quarter of the entire cohort) added the numerators and/or denominators, an error referred to as the "add across" error (Smith, 2002). The fact that the answer many students gave was less than one indicates that they were not thinking about the size of the individual fractions and the likely impact on the combination of the two fractions. With $\frac{7}{8}$ so close to one, and $\frac{3}{4}$ considerably more than $\frac{1}{2}$, it should have been obvious that the correct answer would be greater than one (Reys, Kim, & Bay, 1999). This finding points to the value of using benchmarks as reference points (e.g., 0, $\frac{1}{2}$, 1) as a way to help students appreciate the magnitude of particular fractions (Reys et al., 1999; van de Walle, 2004).

It was interesting to note that only a relatively small number of students drew diagrams to help them solve the pizza problem (26, of whom five gave a correct response, 18 gave an incorrect response, and three made no attempt). Only five of the students who got a correct answer used a diagram to help them, perhaps because many were able to solve the problem using abstraction. Although 18 of the students who produced an incorrect response drew diagrams, these were not always helpful. Three students drew diagrams but were unable to connect their intuitive understanding of fractions, as reflected in their diagram, with the formal written symbolism they had been taught at school. Like other writers, we believe that diagrams, as well as other physical materials, have much to offer in helping students to make sense of the problem by using pictorial representation (see Lamon, 2007, van de Walle, 2004). In our discussions with students about their perspectives on their mathematics learning, we got a clear impression that many of them viewed the use of physical materials as appropriate only for younger students or for students experiencing major difficulties with mathematics. It was interesting to note that the students who did use diagrams all used circular diagrams to depict fractions. Several writers (for example, Bay, 2001) have warned that the over-reliance on drawing pictures of pies may impede the development of a more abstract understanding of what a fraction is and thus slow down the acquisition of "the basic disposition towards algebra" (Wu, 2002, p. 60).

It is important to acknowledge that caution should be exercised in drawing firm conclusions about a student's conceptual understanding from just one task, as Mitchell and Clarke (2004) have pointed out. However, the use of a familiar context such as pizzas divided into quarters and eighths ought to have given the students the best possible chance to show any understanding of fractions that they did have. We recognise that there are other tasks that could provide insights about other aspects of students' understanding of fractional numbers. However, the advantages of being able to audiotape the entire conversation with each student in order to capture their verbal explanations as well as any written recording they may have done have helped to provide a rich source of data about students' thinking and problem-solving processes. The size of the cohort (n = 238), and the fact that it includes a substantial number of Māori and Pasifika students means that this data set may be able to provide some answers to many important questions, not just about students' mathematical thinking but also how that is related to their perspectives and views about their mathematics learning at school and beyond.

The findings of this study underline the importance of working towards ensuring that students having a deep and connected understanding of fractions, beginning this process from the earliest years at primary school. It is important for teachers to recognise the importance of their own subject-matter knowledge of mathematics and its impact on the learning of their students and to take responsibility for addressing their own learning needs in this domain. The strategies identified in this study provide a useful starting point for teachers in relation to pedagogical content knowledge. For example, the "make a whole" strategy could provide a useful alternative to other more conventional methods of solving a problem involving addition of fractions. This could be part of an approach that capitalises on familiar elements of the Number Framework currently used to develop students' understanding of whole numbers (see Mack, 1990; van de Walle, 2004). For example, students could be shown how to *count* with fractions, beginning with the easiest and most familiar fraction, $\frac{1}{2}$, and using cardboard semi-circles to model the counting process, as in $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, and so on. This might be following by counting verbally without materials (that is, *imaging*). This experience with counting might help students to appreciate that they can use the same processes to count units that are fractional parts as they already use to count units of one or units that are multiples, such as fives or tens. Progressing to other fractions such as $\frac{1}{4}$ would allow fraction equivalence to be experienced within the context of counting, as in $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1, $1\frac{1}{4}$, $1\frac{1}{2}$, $1\frac{3}{4}$, 2, and so on. This could be followed by adding fractions, initially with sums within a whole, but later with sums beyond a whole, using the "make a whole" strategy as a way of using knowledge of partitioning to break the addition process into steps; first making one of the fractions into a whole by joining it with part of the other fraction, then adding the remaining fractional part. *Compensation* strategies could be modelled, as in $\frac{3}{4} + \frac{7}{8}$ is the same as $\frac{3}{4} + (1 - \frac{1}{8})$, so $1\frac{3}{4} = 1\frac{6}{8}$ and $1\frac{6}{8} - \frac{1}{8} = 1\frac{5}{8}$. *Doubling* and *halving* strategies could also be explored, as in $1\frac{1}{2} + 1\frac{1}{2} = 3$, or half of $1\frac{1}{2}$ is $\frac{1}{2} + \frac{1}{4}$, which is the same as $\frac{3}{4}$. A similar approach could be used with subtraction, starting with counting back by fractional units, then using "bridging through a whole" (similar to "bridging through ten"; see Thompson, 1999, 2000) to subtract across wholes, as in $1\frac{1}{4} - \frac{5}{8}$ is the same as $1\frac{2}{8} - (\frac{2}{8} + \frac{3}{8}) = 1 - \frac{3}{8}$ and $\frac{8}{8} - \frac{3}{8} = \frac{5}{8}$. By drawing on some of the key processes that are used in building whole number understanding, teachers would be helped to appreciate some commonalities between the whole-number system and the rational-number system. This kind of approach might also help students to develop flexibility in unitising and reunitising quantities (Lamon, 2007). (Note: Lamon uses the term *unitising* (p. 630) to refer to "the process of mentally chunking or restructuring a given quantity into familiar or manageable conveniently-sized pieces in order to operate with that quantity"). The ideas suggested above would strengthen the measurement sub-construct for fractions, the sub-construct Lamon believes provides one of the best starting points for building understanding of rational numbers.

It is important to acknowledge that *time* is a key issue in coming to understand fractions. According to Lamon (2007), "multiplicative ideas, in particular, fractions, ratios, and proportions, are difficult and develop over time" (p. 651). This is supported by her research findings, that in the longer term, it was the deep and connected understanding of fractions acquired by students in the experimental classrooms that provided them with the power and flexibility to perform meaningful operations and eventually to surpass the rote learners. Lamon's research findings underline the importance for educators of being patient but persistent in bringing about change in the teaching of fractions. It also supports the call for longer and more sustained professional development for teachers working with middle-years students, who need to become multiplicative thinkers if they are to engage productively with algebra at secondary school.

The challenge for us now and in the future is to ensure that students do not give up the search for sense making and understanding in mathematics or turn to procedural approaches for solving problems (van de Walle, 2004). However, this is no mean feat. It requires a major shift for teachers in ways of thinking about the goals of mathematics learning. The emphasis must be on building conceptual understanding at all levels of the school. Fractions provide an ideal context in which to take on this challenge.

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Numeracy Sustainability: Current Initiatives and Future Professional Development Needs

Jenny Ward
Maths Technology Ltd.
<jenny@nzmaths.co.nz>

Gill Thomas
Maths Technology Ltd.
<gill@nzmaths.co.nz>

Andrew Tagg
Maths Technology Ltd.
<andrew@nzmaths.co.nz>

As the implementation phase of the Numeracy Development Projects (NDP) moves into its final year, the ongoing success of the projects will depend on schools' ability to take more responsibility for developing and maintaining effective numeracy practices. This paper examines the views of lead teachers and facilitators in schools that have been involved in the NDP since its inception and outlines their views on the current sustainability initiatives and future professional development needs of schools and teachers. The extent and effectiveness of current initiatives in sustainability were found to be varied. Facilitators identified school-wide factors such as the provision of release time for lead teachers to support other teachers as most helpful for sustaining and developing effective numeracy teaching. Lead teachers reported classroom-focused factors such as the provision of quality resources as key to sustaining effective numeracy practices. Lead teachers and facilitators agreed on the need to develop teacher content knowledge, especially in the upper stages of the Number Framework, the need to support provisionally registered teachers, and the need to update the training of those schools and teachers that trained in the initial years of the NDP.

Background

The Numeracy Development Projects (NDP) began in New Zealand in 2000 with the successful implementation of the Count Me In Too pilot project. Since then, more than 17 000 teachers and 460 000 students have participated in the projects, and by 2007, almost all teachers of students in years 1–8 will have had the opportunity to take part (Parsons, 2005). The NDP were conceived as dynamic and responsive to research from their outset, and, as they have developed, they have been continually informed by ongoing evaluations. Now, as the projects move out of their initial implementation phase and into a sustainability phase, the focus of the NDP facilitation contracts will shift to consolidation and maintenance. The challenge will be to maintain the strategic focus, quality, and momentum of the NDP while shifting the ownership for ongoing development to the school level (Thomas & Tagg, 2006).

> The NDP is moving into a phase in which the emphasis is not only on improving the teaching and learning of mathematics in New Zealand schools but also on enhancing the capacity of schools to sustain and build on that learning. (Ministry of Education, 2005, p. 4)

In line with this changing focus, the Ministry of Education has commissioned studies investigating issues of sustainability as part of the ongoing evaluations of the NDP over the past two years (for example, Thomas, Ward, & Tagg, 2005; Thomas & Ward, 2006).

A number of factors have been identified as important for schools in order to sustain and develop effective numeracy practices. The use of student-achievement data as the basis for teachers' critical reflection has been acknowledged as key, with schools who report extensive use of numeracy-achievement data appearing to raise the achievement of their students more than schools who report a lower use of student-achievement information (Thomas & Tagg, 2005). Other factors seen as important in sustaining and developing numeracy include the embedding of numeracy into school-wide documentation and practices, the development and implementation of procedures to upskill teachers new to the school who may lack numeracy training, and the development of effective ways to communicate with parents and caregivers about numeracy developments.

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Jenny Ward
Maths Technology Ltd.
<jenny@nzmaths.co.nz>

Andrew Tagg
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<andrew@nzmaths.co.nz>

Gill Thomas
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All schools that had been involved in the NDP prior to 2006 were invited to participate in this research. In particular, the extent and effectiveness of the sustainability initiatives and the factors seen as fundamental to the sustaining of effective numeracy practices in these schools were examined.

Method

Participants

The sample included all schools that had participated in the NDP initiatives between 2000 and 2005. Regional numeracy co-ordinators were asked to provide lists of schools that had received training up to and including 2005 schools. Schools participating for the first time in 2006 were not surveyed. Surveys were distributed to 1 329 schools, with responses received from 349 lead teachers. It is not possible to calculate an accurate response rate because some schools have more than one lead teacher. It should be noted that this is a relatively low response rate, and it is not possible to determine the extent to which responses are representative of the population.

Numeracy facilitators working with schools in sustainability initiatives in 2006 were also targeted for feedback, with regional numeracy co-ordinators providing lists of these facilitators. A total of 62 facilitators were sent surveys; responses were received from 38 facilitators, giving a response rate of 61%.

Procedure

Surveys were developed to gather information from key participants. This paper reports on the responses received from lead teachers and facilitators. Questions focused on the extent and effectiveness of professional development support received in 2006, participants' professional knowledge, and the future professional development needs of schools. The majority of questions involved closed responses, with participants being asked to rate factors. A small number of questions required participants to provide reasons for their responses or briefly describe key factors and ideas.

Emails were sent to schools (addressed to lead teachers) and facilitators in early November, asking them to complete the survey online, with returns requested one week later. The email contained instructions on how to complete the survey and a hyperlink to the survey. An email was sent to schools and facilitators in the middle of November reminding them to complete the surveys, with regional co-ordinators also being asked to remind participants to complete the surveys at this time. All surveys completed prior to November 25 were included in the evaluation.

Findings

Two key research questions were investigated. These were: "To what extent are the sustainability initiatives meeting the professional learning needs of individual teachers?" and "What elements of numeracy support are needed in order to sustain or further develop effective numeracy teaching and learning in schools?"

This section describes the findings of the study under four key headings: extent of support, effectiveness of support, professional knowledge, and future needs. Comments from participants have been used to illustrate themes and are taken directly from surveys. Where percentages do not add to 100, this is due to rounding error.

Extent of Support

Facilitators were asked to report how many schools they had worked with in order to sustain numeracy practices in 2006. They were also asked to identify how many of these schools were provided with in-depth support. Table 1 shows these results.

Table 1
Numbers of Schools Supported by Facilitators

	Number of facilitators	
Number of schools	In-depth support	Other support
Less than 5	23	10
6–10	11	7
11–20	4	10
More than 20	0	11

In total, the facilitators who responded provided 106 schools with in-depth numeracy support and 787 schools with other support in numeracy. On average, each facilitator supported five schools in-depth and 23 other schools. There was a large range in the number of schools supported by each facilitator, with the number of schools supported in-depth ranging from 0–19 and the number of other schools supported ranging from 1–185.

Facilitators identified a variety of people as being responsible for selecting schools for in-depth support. These were the regional numeracy co-ordinators, the numeracy advisory teams, and management staff from School Support Services. It was acknowledged that staff from other advisory services, in particular, their leadership and management advisors, were often consulted when selecting schools. Lead teachers reported receiving support from facilitators in a variety of ways. Eighty-four percent of lead teachers attended facilitator-run workshops, with 26% of lead teachers spending 5–10 hours in workshops and 25% spending more than 10 hours. Lead teachers also reported receiving support via email and phone (69%) and from facilitator visits to their school (64%). A small number of other support mechanisms were reported, with the one common theme being facilitator assistance to run an information evening for parents and caregivers about the NDP.

> A facilitator visited our school to lead a successful parents' evening.
>
> [Name] was at a parent evening with us and I personally found this supportive and helpful.

The focus of the support received by lead teachers varied. Table 2 shows the extent of support in a number of key areas, as reported by lead teachers.

Table 2
Extent of Support Received by Lead Teachers

	No help required	None	Minimal	Moderate	Extensive
Establishing targets	12%	28%	23%	27%	11%
Developing a school-wide plan for numeracy sustainability	8%	29%	20%	30%	14%
Establishing school-wide data collection systems	10%	31%	22%	26%	12%
Implementing peer observations of numeracy teaching	10%	46%	21%	16%	7%
Leading staff meetings based on numeracy teaching and learning	8%	35%	19%	24%	14%
Working with teachers new to the school	11%	28%	18%	28%	16%
Providing in-class mentoring and support for teachers	7%	38%	18%	24%	13%
Providing informal support for teachers in numeracy	4%	24%	25%	32%	15%

Forty-seven percent of the lead teachers reported receiving moderate to extensive support with providing informal support for teachers in numeracy, and 44% of lead teachers reported receiving moderate to extensive support in developing a school-wide plan for numeracy sustainability and working with teachers new to the school. The least support was reported in the area of implementing peer observations of numeracy, with 46% of the lead teachers reporting no support in this area. Other areas in which lead teachers reported receiving no support were providing in-class mentoring and support for teachers (38%) and leading staff meetings based on numeracy teaching and learning (35%). When asked to identify other areas of numeracy in which they had received support, lead teacher responses were varied but tended to focus on teaching resources.

> Supplied with latest numeracy info, support to find my way around the nzmaths site, good ideas from other teachers co-ordinated.
>
> Guidance with setting up school-wide resources and accessing appropriate websites.
>
> How to use maths equipment. The right type of maths equipment to buy.

Effectiveness of Support

Lead teachers were questioned on the extent to which their professional learning needs were met, both as a lead teacher and as an individual. These results are shown in Table 3.

Table 3
Extent to Which Professional Learning Needs Were Met

	Not addressed	Partially met	Met	Fully met
Learning needs as a lead teacher	10%	37%	40%	13%
Learning needs as a classroom teacher	10%	30%	44%	16%

In general, lead teachers report their professional learning needs as a classroom teacher as being more fully met than their professional learning needs as a lead teacher. Sixty percent of lead teachers report their needs as an individual as either met or fully met, while 53% report having their learning needs as a lead teacher either met or fully met. Ten percent of lead teachers identified that their needs as a lead teacher and as a classroom teacher were not addressed.

Comments from those lead teachers who felt their needs were being met or fully met described the effectiveness and availability of the facilitator.

> Facilitator was approachable and well informed and could address any issue.
>
> Advisors always replied promptly to any requests or queries I had. This was greatly appreciated.

Comments from those lead teachers who identified their professional learning needs as either partially met or not addressed tended to reflect these teachers' feelings of isolation, resulting from difficulties in accessing support. Often, these lead teachers had inherited the role of lead teacher for numeracy from another teacher and therefore missed the initial lead-teacher training.

> At times it was very difficult to contact facilitators, as no actual facilitator appeared to be assigned to us. Teachers who were on courses sometimes came back to school with concepts/ideas that were in contrast to those that we were guided towards during our training. I realise that things change over time but have found it difficult to keep "updated".
>
> As a new lead teacher, I have felt that I have had to fend for myself a bit. It has been a challenge to keep up with new initiatives and stay on top of new developments. I have often found out about new resources by accident rather than being told about them by facilitator.

In the end-of-year survey, lead teachers were asked to rate their schools on a number of numeracy practices as at the start and end of 2006. These results are presented in Table 4.

Table 4
Lead Teachers' Ratings of Numeracy Practices at the Start and End of 2006

		None	Under-developed	Established	Well established
Student-achievement targets	Start	10%	35%	38%	17%
	End	3%	29%	45%	23%
School-wide plan for numeracy sustainability	Start	15%	42%	33%	9%
	End	5%	36%	42%	17%
School-wide data collection systems	Start	7%	38%	36%	20%
	End	2%	27%	41%	30%
Staff meetings based on numeracy teaching and learning	Start	19%	46%	28%	7%
	End	13%	37%	40%	10%
Involving and informing families	Start	27%	44%	25%	4%
	End	17%	39%	35%	8%
Working with teachers new to the school	Start	23%	33%	36%	8%
	End	15%	23%	46%	16%
In-class mentoring and support for teachers	Start	23%	42%	31%	4%
	End	17%	34%	41%	7%
Informal support for teachers in numeracy	Start	8%	30%	52%	10%
	End	4%	22%	58%	17%
Teaching basic facts	Start	3%	29%	45%	23%
	End	1%	14%	54%	31%
Teaching additive strategies	Start	5%	25%	51%	20%
	End	1%	9%	60%	30%
Teaching multiplicative strategies	Start	6%	29%	50%	14%
	End	2%	15%	62%	22%
Teaching proportional strategies	Start	9%	40%	41%	10%
	End	4%	26%	56%	14%
Teaching effective written recording of solution strategies	Start	12%	44%	36%	8%
	End	6%	34%	50%	10%

Lead-teacher ratings at the beginning of the year indicate the most well-established numeracy practices were the teaching of basic facts, with 23% of lead teachers reporting this as well established within their school. Twenty percent of lead teachers reported the teaching of additive strategies and the implementation of school-wide data-collection systems as well established. In contrast to these more well-established practices, 27% of lead teachers rated their school as having no way of involving and informing families about numeracy teaching and learning at the start of 2006. Twenty-three percent of lead teachers also identified that their schools had no processes for working with teachers new to the school or for mentoring and supporting teachers in-class at the start of 2006.

As at the end of 2006, the most well-established numeracy practices were the teaching of basic facts, additive strategies, and the implementation of school-wide data-collection systems. There was an increase in the percentage of lead teachers rating their school as well established in these practices by the end of 2006. Thirty-one percent of lead teachers rated their schools as well established in the teaching of basic facts, 30% rated their schools as well established in the teaching of additive strategies, and 30% had well-established school-wide data collection systems.

In comparison with start of the year ratings, the least well-established numeracy practices at the end of 2006 were the involvement and informing of families, working with teachers new to the school, and the mentoring and supporting of teachers in class. There was a decrease in the percentages of lead teachers rating their schools as having no organisation in these areas. By the end of 2006, the percentage of lead teachers identifying their schools as having no means of involving and informing families had decreased from 27% to 17% and the percentage of lead teachers rating their school as having no developed way to support teachers new to the school decreased similarly, from 23% to 15%. The percentage of lead teachers identifying their schools as having no established systems for in-class mentoring and support dropped from 17% to 23%.

These lead-teacher ratings of their school's numeracy practices can be used as a measure of progress. Table 5 summarises the changes in lead teachers' ratings for each of the numeracy practices. The numbers given show the percentages of lead teachers reporting improvements and declines in each area. The numbers in brackets give the percentages of lead teachers that have made a shift, excluding those that had no possibility of shifting. For example, 30% of all lead teachers reported an improvement in their school's use of student-achievement targets; however, when those schools that initially rated themselves as well established are excluded, the percentage increases to 36%.

Table 5
Percentage of Lead Teachers Reporting Changes in Numeracy Practices

	Improvement	Decline
Student-achievement targets	30 (36)	4 (5)
School-wide plan for numeracy sustainability	34 (38)	5 (4)
School-wide data collection systems	29 (37)	2 (2)
Staff meetings based on numeracy teaching and learning	26 (27)	5 (6)
Involving and informing families	27 (28)	1 (2)
Working with teachers new to the school	31 (33)	2 (3)
In-class mentoring and support for teachers	22 (23)	1 (1)
Informal support for teachers in numeracy	22 (24)	1 (1)
Teaching basic facts	26 (34)	1 (1)
Teaching additive strategies	29 (36)	1 (1)
Teaching multiplicative strategies	28 (33)	1 (1)
Teaching proportional strategies	26 (29)	1 (1)
Teaching effective written recording of solution strategies	22 (24)	1 (1)

Note: Percentages in brackets exclude those teachers unable to change.

The most progress was seen in the areas of school-wide plans for numeracy sustainability, school-wide data-collection systems, use of student-achievement targets, and the teaching of additive strategies. Over one-third of lead teachers (excluding those that rated their school as well developed at the start of 2006) reported improvements in their school in these areas. The least progress was seen in the areas of in-class mentoring, the provision of informal support for teachers, and the teaching of effective written recording of solution strategies. In these areas, nearly one-quarter of lead teachers (excluding those that rated their school as well developed at the start of 2006) reported improvements.

Facilitators were asked to describe how they determined if schools were becoming more effective in their numeracy practices. A variety of factors were identified as important, with the use of student-achievement data being one of the most frequently noted.

> Examining "hard data". For example, one decile two school has reduced their percentage of year 6 students at stage 4 from around 65% to 12%.
>
> Tracking data over a 2 or 3-year period.

Classroom observation and discussions with staff were also identified as evidence of the increasing effectiveness of schools' numeracy practices. In particular, teachers showing the ability to modify and adapt lessons and teaching approaches to meet the needs of their students, an increase in reflective comments made by teachers, and an increasing professional dialogue between staff about mathematics and student achievement were all noted as significant.

> By observations/discussions with myself, lead teachers, and principals. Note deeper engagement by teachers with what they are trying to achieve. This is evidenced by a shift from a focus on resources ("tell us a new game") to learning.
>
> Increased teacher professional conversations. Increased teacher goal setting and critical buddies for observations.
>
> Teachers can pick up any resource and adapt the learning to suit the teaching model.
>
> The level of conversation about maths learning and student achievement increases. They follow you out to the car to ask you more questions. There is consistent delivery of numeracy core practices.

Professional Knowledge

Lead teachers and facilitators were given a set of statements about numeracy teaching and learning and asked to indicate their level of agreement with each of these. Facilitators and lead teachers held similar views on approximately three-quarters of the statements, with both agreeing on the importance of regularly reviewing students' progress in the light of achievement targets, the idea that student groupings should be flexible, change as required in response to classroom observation, and the importance of developing instant recall of basic facts once these are understood.

Lead teachers and facilitators held differing views on three of the statements. These are shown in Table 6.

Table 6
Differing Professional Views Between Facilitators and Lead Teachers

	Lead-teacher view		Facilitator view	
	Agree	Disagree	Agree	Disagree
It is important to teach numeracy lessons exactly as they were planned	29%	72%	5%	95%
It is important to carry out a full diagnostic interview with all students at least once a year	46%	54%	5%	95%
It is important to have a continual supply of new resources for numeracy	67%	33%	32%	68%

Forty-six percent of the lead teachers agreed that it is important to carry out a full diagnostic interview with all students at least once a year, while just 5% of facilitators are in agreement. This is an interesting finding because the time taken to carry out a full diagnostic assessment for all students creates resourcing issues for schools that can hinder the sustainability of effective numeracy practices (Thomas, Tagg, & Ward, 2006).

Sixty-seven percent of the lead teachers agreed that it is important to have a continual supply of new resources for numeracy, while 32% of facilitators agreed that this is important. This importance placed by teachers on resources is in accordance with previous findings (Thomas & Ward, 2006) and reflects the high priority that teachers place on ongoing teaching resources in numeracy.

Future Needs

Lead teachers and facilitators were asked to rate a number of factors according to their helpfulness for sustaining and developing numeracy. These results are shown in Table 7.

Table 7
Percentages of Lead Teachers and Facilitators Rating Factors As Most Helpful

		Not helpful	Helpful	Very helpful	Essential
Ongoing facilitator support of lead teachers	Lead teacher	1	22	28	49
	Facilitator		5	21	74
In-class monitoring and use of student-achievement data	Lead teacher	1	22	29	48
	Facilitator		11	29	61
Principals' participation in numeracy developments and direction within the school	Lead teacher	2	28	28	42
	Facilitator		3	21	76
Regular syndicate or school-wide review of student achievement using achievement targets	Lead teacher	1	22	40	37
	Facilitator		8	32	61
Release time provided for lead teacher to support other teachers	Lead teacher	2	15	34	49
	Facilitator		3	8	90
Release time provided for teachers to assess students	Lead teacher	1	15	32	52
	Facilitator	3	45	37	16
Material on nzmaths website	Lead teacher	12	35	54	
	Facilitator		8	29	63
Access to further qualifications in mathematics teaching	Lead teacher	10	39	40	12
	Facilitator		16	58	26

In general, facilitators indicated they regard these factors as more helpful in sustaining and developing numeracy than lead teachers, with facilitator ratings more positive than lead teachers' ratings for all factors. Facilitators identified the most helpful factor for sustaining and developing numeracy as the provision of release time for lead teachers to support other teachers, with 90% of facilitators identifying this as essential. In comparison, 49% of lead teachers believed this was essential to sustaining numeracy.

> Adequate financing available to support the release lead of lead teachers and allow for peer observations by teachers. (Facilitator)

Other factors rated highly by facilitators were the principals' participation in numeracy developments and direction within the school, regarded as essential by 76% of facilitators, and ongoing facilitator support of lead teachers, identified as essential by 74% of facilitators.

In contrast to the facilitators' views, lead teachers identified the most helpful factor for sustaining and developing numeracy as the material on the nzmaths website, with 54% of the lead teachers regarding this as essential.

> I have found that most of my information has come from the nzmaths site, which has been a fantastic resource.

Lead teachers also regarded the provision of release time to assess students as helpful, with 52% of lead teachers rating this as essential.

Lead teachers were asked to identify the three most important processes implemented in their school that contribute to sustaining effective numeracy practices. Responses were open-ended, and results were categorised and grouped to find common themes. These results are shown in Table 8.

Table 8
Lead Teacher Views on Most Important Processes for Sustaining Numeracy Practices

Processes	Percentage of lead teachers
Resources	34
Student-achievement data – collection and use	26
Meetings focused on numeracy teaching and learning	24
School-wide plans and processes	23
Teacher collaboration	18
Ongoing access to professional development for all staff	15
Effective assessment tools	13
Student-achievement targets	11
Informal/ongoing support facilitator	10

As is apparent, not all of the factors identified were processes, but the responses give a valid indication of lead teachers' views on factors within their school that contribute to the sustainability of numeracy developments. Thirty-four percent of lead teachers identified the supply and organisation of resources within the school as contributing to sustaining effective numeracy practices. The collection and use of student-achievement data and staff and syndicate meetings focused on numeracy teaching and learning were identified by 26% and 24% of lead teachers respectively.

> ... meetings where everyone brings an activity that has worked well for them or a resource they can't get the hang of and others suggest ways to use it.
>
> Providing Numeracy Trolleys specially designed to store the necessary equipment for easy access for teachers.
>
> Collection of student-achievement data to enable school-wide target setting. Ongoing in-class monitoring.

Twenty-three percent of lead teachers regarded the implementation of school-wide plans and processes as a key factor in the sustainability of numeracy practices. A wide variety of school documents and procedures were identified, including mathematics curriculum plans, plans for numeracy sustainability, long-term plans, and co-operative planning and assessment practices.

Facilitators were asked to rate the lead teachers in their region in terms of their future professional development needs in a number of areas. Results are presented in Table 9.

Table 9
Facilitator Views on Lead Teachers' Professional Development Needs

	Minimal	Moderate	Extensive
Teaching proportional strategies	63%	34%	3%
Working with teachers new to the school	55%	37%	8%
Peer observations of numeracy teaching	53%	34%	13%
Teaching multiplicative strategies	47%	42%	11%
In-class mentoring and support for teachers	47%	45%	8%
School-wide plan for numeracy sustainability	45%	47%	8%
Involving and informing families	24%	58%	18%
Staff meetings based on numeracy teaching and learning	11%	58%	32%

Facilitators identified the most important professional development need for lead teachers in the future was to develop staff meetings based on numeracy teaching and learning, with 32% of facilitators rating lead-teacher needs as extensive in this area. Other factors identified as important by facilitators were involving and informing families and peer observations of numeracy teaching, identified as extensive needs by 18% and 13% of facilitators respectively.

When asked to identify other professional development required in order to sustain and develop effective numeracy teaching, three common themes emerged from the responses of both lead teachers and facilitators. These were the need to develop teacher content knowledge, especially in the upper stages of the Number Framework, the need to support provisionally registered teachers, and the need to update the training of those schools and teachers that trained in the initial years of the NDP.

> Improving their own pedagogical content knowledge so they can effectively model for teachers in their school. (Facilitator)
>
> As we are a large school and constantly have new staff and beginning teachers, there should be an ongoing cycle of PD provided to meet their needs, funded by MOE, incorporated in beginning teacher PD. (Lead teacher)
>
> A refresher course from facilitators for the schools and teachers that have been part of the project from the beginning. There have been many changes and it would be good to get refocused to carry on. (Lead teacher)

Concluding Comments

Lead teachers reported receiving support from facilitators in a variety of ways. Although the scope of the support received was also varied, areas of focus identified were the successful provision of informal support for teachers in numeracy, the development of a school-wide plan for numeracy sustainability, and effective ways to work with teachers new to the school.

In general, lead teachers reported their professional learning needs as a classroom teacher as being more fully met than their professional learning needs as a lead teacher. Lead teachers reported the most well-established elements of numeracy practices as the teaching of basic facts, the teaching of additive strategies, and the implementation of school-wide data-collection systems. The least well-established numeracy practices were reported as the involvement of families and informing them about numeracy, working with teachers new to the school, and in-class mentoring and support for teachers. The most progress over the year occurred in the areas of developing school-wide plans for numeracy sustainability and data-collection systems, the use of student-achievement targets, and the teaching of additive strategies.

In general, lead teachers and facilitators reported similar professional views on numeracy practices. However, lead teachers placed a higher importance on carrying out a full diagnostic interview regularly with all students and the provision of a continual supply of new resources.

Facilitators identified school-wide factors such as the provision of release time for lead teachers to support other teachers, the principals' participation in numeracy developments, and ongoing facilitator support of lead teachers as most helpful for sustaining and developing numeracy. In contrast to this, lead teachers identified classroom-focused factors such as the material on the nzmaths website, the provision of release time to assess students, and the provision and organisation of teaching resources as most helpful. Lead teachers and facilitators agreed on the need to develop teacher content knowledge, especially in the higher stages of the Framework, the need to support provisionally registered teachers, and the need to update the training of those schools and teachers that trained in the initial years of the NDP.

It is clear that progress was made in each of the areas of numeracy development identified, with between 24–38% of lead teachers noting an improvement in their school for each factor. However, more than a third of lead teachers felt that their learning needs were either not addressed or only partially met, both as a lead teacher and as a classroom teacher of numeracy. In addition, the factors that were seen by lead teachers as most helpful in sustaining and developing numeracy practices were not always consistent with those identified as most important by facilitators.

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Leading a Curriculum Reform from Inside a School

Joanna Higgins
Victoria University of Wellington
<joanna.higgins@vuw.ac.nz>

Brenda Sherley
Victoria University of Wellington
<brenda.sherley@vuw.ac.nz>

Sandi Tait-McCutcheon
Victoria University of Wellington
<sandi.mccutcheon@vuw.ac.nz>

This case study examines which domains of knowledge underpin effective lead teacher practices that develop teacher capacity and increase student learning. The main focus was on identifying the domains of knowledge perceived by lead teachers themselves, principals, and teachers as critical to effective leadership practice. Four domains of knowledge perceived to be important were knowledge of, and attitude towards, mathematics, knowledge of students as learners, knowledge of teachers as learners, and knowledge of communities as learners.

Background

The central focus of the New Zealand Numeracy Development Projects (NDP) is to raise student achievement in mathematics by improving the professional capability and capacity of teachers across all New Zealand schools. The NDP began as a pilot study in 2000 in response to the poor performance of New Zealand students in the Third International Mathematics and Science Study (TIMSS) (Garden, 1997; see Higgins, Parsons, & Hyland, 2003).

As the NDP has progressed, the emphasis has shifted as a result of regular evaluations commissioned by the Ministry of Education[1].

> The NDP is moving into a phase in which the emphasis is not only on improving the teaching and learning of mathematics in New Zealand schools but also on enhancing the capacity of the schools to sustain and build on that learning. (Ministry of Education, 2005, p. 4)

In 2004 and 2005, the Ministry commissioned evaluations that looked specifically at sustaining practice in schools (for example, Thomas & Tagg, 2004; Thomas & Ward, 2005[2]). In response to the need to develop in-school sustainability, there was an increased focus on the lead teacher component of the NDP professional development. In a 2005 case study, Thomas and Ward (2006) found that some participants in their research appeared to misunderstand the scope of the programme by viewing the development as the workshop component only. These participants had a narrow view of professional development and believed that on-going in-depth external support was required for them to sustain the development. In response to Thomas and Ward's finding, the case study reported on here specifically investigated perceptions of leadership content knowledge as they applied to the NDP.

A school-based lead teacher approach to professional development has been less frequently used in New Zealand schools compared with the more usual externally imposed "one-size-fits-all" model of professional development typically implemented through a design adherence approach (Higgins, 2005). In the lead teacher initiative of the NDP, the external facilitator's role is to support the lead teacher in developing their knowledge and professional practice in the context of their school setting.

As well as undertaking administrative tasks, lead teachers are responsible for liaison and communication within the school community while also performing a significant professional development role. The

1 The reports can be retrieved from www.tki.org.nz/r/literacy_numeracy/litnum_research_e.php

2 The latter report found that lead teachers had increasing confidence in leading professional practice within their schools.

Ministry of Education website states that the personal qualities necessary for lead teachers include enthusiasm and interest in mathematics and the ability to effectively support colleagues (Ministry of Education, 2006). Goleman, Boyatzis, and McKee (2002) defined resonant leaders as being "in sync" with their colleagues and having a high level of emotional intelligence that enables them to form "an emotional bond that helps them stay focused even amongst profound change and uncertainty" (p. 21).

This current case study examines the perceptions that facilitators, lead teachers, principals, and teachers have of the knowledge underpinning effective lead teacher practices.

Rationale

As researchers, we must ask *What domains of knowledge inform leadership actions that shift teacher practice and enhance student outcomes?* A study of leadership knowledge provides a frame for examining the leadership that is occurring, how it is applied, and why it takes the particular forms it does. Research on leadership practices clarifies what leaders actually do, and why. Research on teaching and learning provides important clues about which practices are likely to make a difference (Robinson, 2004, p. 41) and focuses on what leaders do as well as on what they think about what they do (Stein & Nelson, 2003).

Within the school community, it is important that there is a shared understanding of the lead teacher's role. Without a shared understanding of leadership, the emphasis can be deflected from a focus on leadership of the NDP to administrative tasks. Elmore (2000) argues that "leadership is the guidance and direction of instructional improvement" (p. 13). The leadership goal is not only to develop a vision, build a good relationship within the school community, and manage the school or department efficiently, but also to do all those things in a manner that improves teaching and learning (Robinson, 2004, p. 40).

Knowledge of leadership moves beyond rhetoric about leadership styles, in which a leader's personal attributes are used to judge their effectiveness, to valuing domains of knowledge underpinning leadership practice. If discussions about leadership are restricted to a leader's style, then we run the risk of making the assumption that the style holds constant across different situations; despite years of research, no conclusions have been made about the effectiveness of leaders based on their different styles (Robinson, 2004).

Theoretical Frame

The complexities of leading a curriculum reform from within a school can be understood through viewing the school as an organisational system. Within such a system, there are a number of participants, each with different roles. In this study, we focus on three key members of the school community: the lead teacher, the principal, and the teachers. Together, these participants are part of a community that operates according to a set of rules (explicit and implicit). We were interested in how the elements of the school system transform opportunities for teacher learning. The analysis was guided by previous work that examined the ways in which orientations to professional development varied from those concerned with adhering to the design of the professional development programme to those that attend to the context through emphasising the programme's principles (Higgins, 2005).

The study draws on socio-cultural perspectives, such as those articulated by Wertsch, del Rio, and Alvarez (1995) that suggest a lead teacher's role is to mediate core principles of a project and their enactment in the classroom and the wider school settings. Of particular relevance to this paper are schema relating to professional learning that are classroom-based professional learning and school-based professional community (Higgins & Parsons, 2005).

Shulman (1986) argues that teachers needed a qualitatively different kind of knowledge that would enable them to help others learn. This knowledge was defined as pedagogical content knowledge (PCK), that is, knowledge of ways to represent and explain a subject to make it comprehensible and knowledge of the thinking that students bring to the learning of a subject that makes it easy or difficult to learn.

Stein and Nelson (2003) contend that leaders need a qualitatively different kind of knowledge that will enable them to lead. They believe that leadership content knowledge is required for effective instructional leaders to improve teaching and learning in their schools. Leadership content knowledge is described as "standing at the intersection of subject matter knowledge and the practices that define leadership" (p. 424). Leadership content knowledge enables curriculum leaders and principals to recognise strong instruction when they see it, to encourage it when they do not, and to set the conditions for continuous academic learning among their staff.

Method

Participants

This case study investigated the lead teacher component of a system-wide project (NDP) begun in 2000, which, to date, has involved over 25,000 primary teachers in New Zealand. This specific study focused on 28 lead teachers, 21 principals, 106 teachers, and three facilitators working in 21 schools across three urban areas in the North Island of New Zealand. Each school was completing their third year of the NDP professional development, and the lead teachers had varying degrees of experience in leadership roles as well as a range of experience with the NDP. Table 1 sets out the number of schools approached and how many accepted the offer to be part of the study. (The reasons given by schools for declining to participate in this research included changes in principals and lead teachers and schools feeling that they were already overloaded in terms of professional development and additional research requests.) The table also includes the numbers of lead teachers and principals interviewed and the number of teacher surveys returned. The total number of participants in the study was 158. The data from the facilitators is not included in the later tables presented but has been used to confirm the findings from the school community participant groups (n = 155). Online surveys were used to obtain demographic data that showed that most of the participants taught in schools larger than 200 students. Over half these schools were in the upper deciles. Most participants were female, representing a range of teaching experiences. Few participants had any mathematical qualifications.

Although they did not have any previous facilitation experience, most lead teachers had previous lead teacher experience spread across literacy, mathematics, and social studies. About three-quarters of the lead teachers currently held positions of responsibility with management units for curriculum areas other than numeracy.

Table 1
Participants in the Study

	Schools approached (n = 33)	Schools accepted (n = 21)	Percentage of schools accepted	Facilitator interviews (n = 3)	Lead teacher interviews (n = 28)	Principal interviews (n = 21)	Teacher forms survey distributed (n = 258)	Teacher forms survey returned (n = 106)	Percentage of teacher survey forms returned
Region A	19	12	63%	1	13	12	130	55	42%
Region B	7	4	57%	1	6	4	50	19	32%
Region C	7	5	71%	1	9	5	78 [3]	32	41%
Totals	33	21		3	28	21	258	106	

Procedures and analysis

The interview and survey questions were designed to elicit evidence from multiple sources including lead teachers, principals, teachers, and facilitators. The key purpose of the interviews was to investigate the ways in which lead teachers of numeracy developed their professional practice in order to improve their own and other teachers' content and pedagogical knowledge of mathematics. Drawing from the study by Stein and Nelson (2003), the questions focused on the effectiveness of the lead teacher model, the impact of the role on lead teachers' own knowledge and practice, teachers' pedagogy, and the impact of lead teachers on the school community.

Methods of data collection included:

- Online survey
- Face-to-face interviews with lead teachers, principals, and numeracy facilitators
- Postal questionnaires with teachers.

Lead teacher and principal interviews and teacher surveys were reviewed using a content analysis approach to identify recurring themes in the transcripts (Denscombe, 1998). The emerging common themes were cross-checked for consistency and reliability by another researcher, with any differences in classification being resolved through discussion. To give an overall picture of the themes across the three regions, the number of common references made by each participant group was then expressed as a percentage of the total comments.

Findings

Building on the construct of leadership content knowledge for school administrators proposed by Stein and Nelson (2003), four categories emerged from the recurring themes in the data. The four categories of leadership content knowledge for lead teachers are defined as knowledge of, and attitude towards, mathematics; knowledge of students as learners; knowledge of teachers as learners; and knowledge of communities as learners. Each category is discussed separately, using participants' comments to illustrate themes.

Knowledge of, and Attitude towards, Mathematics

The category of knowledge of, and attitude towards, mathematics evolved from references to mathematics content knowledge and disposition towards mathematics. Principals and teachers

[3] One school from region C declined to participate in the teacher survey.

from all regions believed that lead teachers needed excellent content knowledge at all levels of schooling.

> Our lead teacher ... has exceptional content and pedagogical knowledge and is continuously striving to build on this. She makes links with wider community networks, ensures that resources are available, and works very hard to support and educate staff. (Teacher, region A)

By contrast, the lead teachers placed more significance on their enthusiasm and passion for mathematics than they did on their content knowledge.

> Because I have a passion for mathematics, so it's been something that I'm really pleased that they're spending more time on. We are certainly finding that passion is the key to success. (Lead teacher, region C)

Table 2 shows the relative emphases placed on lead teacher knowledge of, and attitude towards, mathematics from the perspectives of lead teachers themselves and of the principals and teachers with whom they are working. Of particular interest is the comparatively lower emphasis placed by lead teachers on their mathematics content knowledge (two references) compared with 16 and 46 references respectively for principals and teachers.

Table 2
References to Lead Teacher Knowledge of, and Attitude towards, Mathematics[4]

	Lead Teachers (n = 28)	Principals (n = 21)	Teachers (n = 106)	Total (n = 155)
Lead teachers' mathematics content knowledge at all levels	2	16	46	64
Lead teachers' enthusiasm and passion for mathematics	10[5]	5	18	33

Knowledge of Students as Learners

Knowledge of students as learners comprises pedagogical knowledge and the promotion of evidence-based practice. Within the area of knowledge of students as learners, about half the principals and teachers indicated that it was important for lead teachers to have excellent pedagogical knowledge at all levels of schooling. Lead teachers did not give the same emphasis to pedagogical knowledge (two references).

> We have had follow-up workshops as a staff to cement some of the knowledge and to refine various planning and teaching approaches. She's helped cement the knowledge that comes from the project and discuss it as it relates to practice. ... We're going through the process of working with staff to refine some of our beliefs and practices around numeracy, and that is still going on at this stage. (Principal, region B)

Almost half the lead teachers and nearly two-thirds of principals placed importance on the need for lead teachers to promote evidence-based practice, but few teachers made reference to lead teachers promoting this source of knowledge about students as learners.

> She directs the teachers to look at the kids, to use the evidence to identify the kids that we need to be concerned about. We collectively talk about it at management and then in their syndicates. This helps to develop our community of learners. (Principal, region A)

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4 It was possible for participants to give more than one answer.

5 Individual participants may have referred to both aspects of knowledge, while other participants may not have commented on either aspect. Therefore it is not possible to show the total number of comments across the categories. A later section shows the numbers of comments across participants for any one category.

Table 3
References to Lead Teacher Knowledge of Students as Learners[6]

	Lead Teachers (n = 28)	Principals (n = 21)	Teachers (n = 106)	Total (n = 155)
Lead teachers' mathematics pedagogical knowledge	2	11	45	58
Lead teachers' promotion of evidence-based teacher practice	12	13	7	32

Knowledge of Teachers as Learners

The domain of knowledge of teachers as learners is made up of four sub-categories: knowledge of contextually responsive practice, respect as a teacher, organisation of personnel, and organisation of resources. Approximately a third of the lead teachers felt that it was important that they were responsive to, and respectful of, individual needs of teachers at their school. A similar proportion felt that it was important to be respected as an expert teacher and have good organisational skills.

> People have been at all different places on the road to learning with numeracy and I've been able to provide the right sort of support to all the teachers as they needed it. (Lead teacher, region B)

> In my role I've learned that it's a continuum – people come onto the continuum at different places, they then move at different speeds. Some people come on right at the beginning and are so enamoured by it all that they just race through, and other people come in halfway and they won't move, and it's taking all of that into consideration. (Lead teacher, region C)

A fifth of the teachers valued a contextually responsive approach from someone they regarded as an expert teacher, and more than half the principals believed that the lead teacher should provide a positive role model as an expert teacher. However, few principals commented directly that the lead teacher tailored their practice to the needs of individual teachers.

A third of the principals indicated that they valued a lead teacher's skills in personnel organisation. None of the principals saw resource management by the lead teacher as a separate issue.

> You've got to have a teacher who's well respected in the school and someone who's acknowledged as being a good practioner themselves. You've got to be organised, respected, and able to walk the talk. (Principal, region A)

A quarter of the teachers rated the organisational skills of the lead teacher as important, as well as valuing their resource management skills.

> The lead teacher must be an expert of numeracy and be a willing role model for observational sessions of best practice. (Teacher, region B)

Table 4
Reference to Lead Teacher Knowledge of Teachers as Learners[7]

	Lead Teachers (n = 28)	Principals (n = 21)	Teachers (n = 106)	Total (n = 155)
Knowledge of contextually responsive practice	9	2	19	30
Respect as a teacher	9	11	16	36
Organisational skills	8	8	27	43
Resource management	0	0	23	23

[6, 7] It was possible for participants to give more than one answer.

Knowledge of Communities as Learners

The domain of knowledge of communities as learners comprises knowledge of relationship building, knowledge of their school, knowledge of developing a community of learners within the school, and knowledge of sustaining an initiative.

Lead teachers' interpersonal skills were ranked as important by about three-quarters of principals and teachers and about two-thirds of lead teachers. These skills were generally described as being approachable, a good listener, and accessible.

> To me the most important thing is the interpersonal skills. If they don't have those, it doesn't matter how good their classroom is, no one will want to go near them. And it doesn't matter what else they have because without those interpersonal skills they will be ineffective. (Principal, region B)
>
> The lead teacher must be friendly, approachable, flexible, supportive and passionate. (Teacher, region C)
>
> I'm very supportive, I have good relationships with people and enthusiasm for the maths that we are doing and a positive attitude that allows me to work easily and enjoyably with people. (Lead teacher, region A)

Principals and lead teachers valued a lead teacher's commitment to developing and sustaining a community of learners through maintaining the momentum and focusing on the needs of the community.

> It is important to me that they are able to contextualise their new learning to our school and to be able to implement new ideas at our school by getting people on board. (Principal, region A)
>
> We regularly share our planning, or things that have worked well, or ask what our next steps might be. I want there to be a school-wide professional learning community where there is openness with information and with skills. (Lead teacher, region A)

However, both principals and lead teachers believed that, as well as being supportive, the lead teachers also needed to take a somewhat hard-line approach and not allow any excuses to get in the way of schools achieving their goals.

> They need to have the ability to make teachers front up. To stand up and say "hey, we need this to happen." They need to be able to motivate others into making a difference. (Principal, region A)
>
> They need to have a balance of patience and impatience, they need to be prepared to nudge people along and to do the hard stuff when people are being resistant or are offering a series of excuses. (Principal, region B)
>
> I need to be able to observe others and critically reflect with them and give honest feedback and advice. Not just being nice but being honest. (Lead teacher, region A)
>
> I haven't allowed anyone to "get off the bus"; I have made sure that I have continually encouraged and supported everyone (and some have needed more encouragement than others) to keep going on the journey. (Lead teacher, region B)
>
> I don't accept "no" from anyone, I'll say "okay, let's see if we can't sort this out together" but I don't allow people to do nothing. (Lead teacher, region C)

Each group valued the lead teacher accepting the role of co-learner within the school community and being a positive role model for colleagues.

> I need to model being a learner, to show that I am also learning and to show that I want to learn. I need to be passionate about the project and the professional development. (Lead teacher, region C)
>
> You've got to be willing to change and model that you are also learning. When you model that, teachers look over and go "oh well, she's prepared to give it a go." (Lead teacher, region C)

Being able to observe and provide effective feedback were practices that were valued by all participants.

> The lead teacher needs to be confident about modelling their own practice and coaching colleagues through theirs. (Teacher, region A)
>
> I would like my lead teachers to be given more time to observe me and give me critical feedback so that my maths programme will continue to improve. (Teacher, region A)
>
> I would like more visits to my classroom so that I could see an effective lesson being modelled and could give and get some feedback. (Teacher, region C)

The NDP facilitators from each region believed that a school-wide focus on the NDP was the key to sustainability. If the NDP is left to develop within syndicates or smaller groups, there is a danger of too many differences and misunderstandings arising. Alignment and synchrony come from school-wide focuses, discussions, and practices.

> Models for planning are explored and refined school-wide. Assessment and reporting procedures have been aligned school-wide to demonstrate learning to teachers, students, and parents. (NDP facilitator, region A)
>
> In-class observations and support have led to increasingly synchronised and sustainable practices. This has come from lead teacher commitment to the whole school. (NDP facilitator, region B)

Table 5
Reference to Lead Teacher Knowledge of Communities as Learners[8]

	Lead Teachers (n = 28)	Principals (n = 21)	Teachers (n = 106)	Total (n = 155)
Knowledge of relationship building	18	17	75	110
Knowledge of developing a community of learners	21	12	29	62
Knowledge of their school	11	5	1	17
Knowledge of the sustaining an initiative	0	4	22	26

Discussion

The construct of leadership content knowledge for lead teachers has evolved from the emphasis that lead teachers, principals, and teachers give to lead teachers' knowledge of, and attitude toward, mathematics, knowledge of students as learners, knowledge of teachers as learners, and knowledge of communities as learners. This section examines the relative importance of the components as perceived by different participants.

Relative Importance of the Components of Lead Teacher Knowledge

The relative importance of the four domains of lead teacher knowledge varies across the participant groups of lead teachers, principals, and teachers as shown in Figure 1. The overall pattern that emerged showed that principals and teachers regarded the knowledge of mathematics more highly than the participating lead teachers. A similar pattern can be seen for importance placed on lead teachers' knowledge of students as learners, with a smaller proportion of lead teachers believing this knowledge was important. Similar proportions of lead teachers, teachers, and principals perceived knowledge of teachers as learners as important. Relatively speaking, it appeared that lead teachers perceived knowledge of communities as learners as the most important knowledge for lead teachers to have.

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[8] It was possible for participants to give more than one answer.

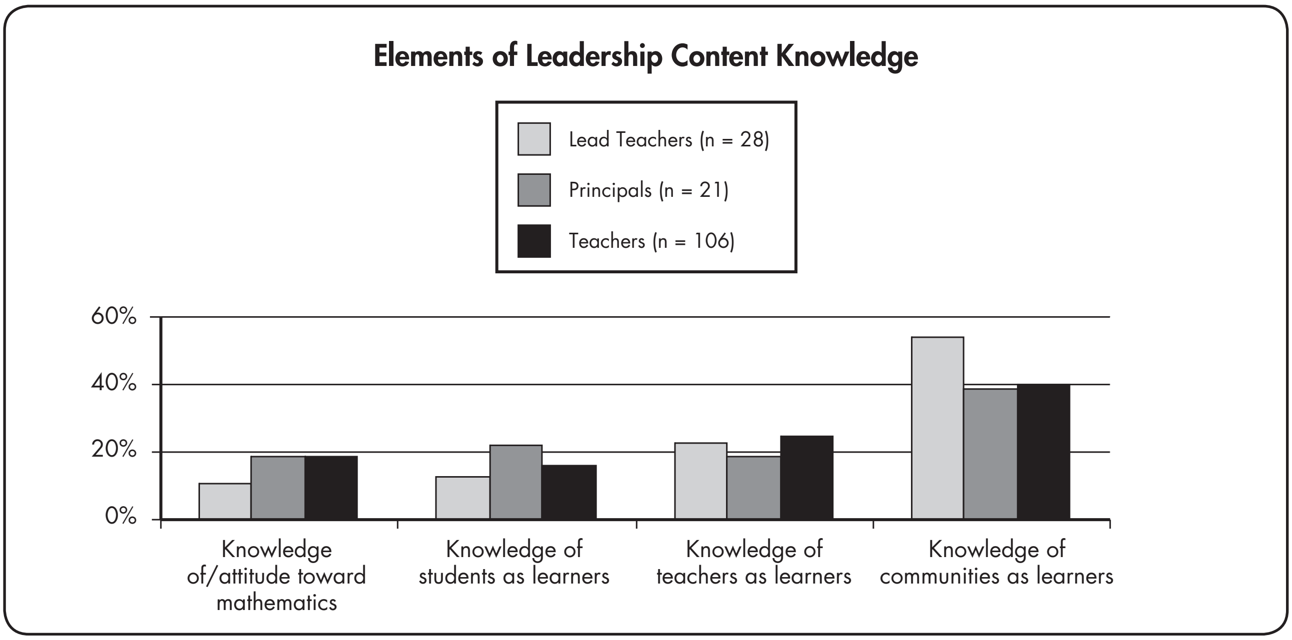

Figure 1: Perceptions by lead teachers, principals, and teachers of the relative importance of components of lead teacher knowledge

The complexities of the interrelationships may be critical to sustaining an initiative through building knowledge of how to foster communities as learners in schools. With this study involving relatively small numbers of geographically-bound lead teachers, further research that investigated the elements of leadership content knowledge would be useful.

> Without knowledge that connects subject matter, learning and teaching to acts of leadership, leadership floats disconnected from the very processes it is designed to govern. (Stein & Nelson, 2003, p. 446)

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Keeping Going at Country School: Sustaining Numeracy Project Practices

Fiona Ell
University of Auckland
<f.ell@auckland.ac.nz>

In this paper, longitudinal data from interviews and videos of classroom practice is used to illustrate the sustainability of Numeracy Development Project (NDP) approaches in a six-teacher rural school. Analysis of interviews with all the teachers in the school reveals a common language and concern about numeracy, which is fostered by ongoing discussions and collegial support. This suggests the emergence of patterns and structures within the school that will allow them to continue to use NDP practices. Consideration of data from 2005 and 2006 reveals the shifts made in both discourse about the NDP and classroom practice. Previously difficult areas have now been internalised, and this has allowed teachers to consider new aspects of their practice. Video data shows the transfer of NDP approaches to strand teaching. Country School continues to embrace, use, and reflect on NDP approaches and students' achievement data, illustrating how a school can develop sustainable practice.

Background

In 2006, Ell and Irwin (2006) reported on the results of a qualitative study undertaken in two schools – City School and Country School. They found that while the schools had taken different paths to implementation – with City School focusing on policy and school-wide structures, while Country School had focused on classroom practice and resources – both schools showed an ongoing commitment to Numeracy Development Project (NDP) practices. A comparison between two teachers, one from each of the two schools, was presented to illustrate how individual internalisation was a key factor in sustaining NDP approaches (Higgins, 2004).

The results of that study and the one reported in this paper serve to elucidate the results found in large-scale questionnaire studies of sustainability (Thomas, Ward, & Tagg, 2005; Thomas & Ward, 2006). Thomas and Ward reported "... a high degree of utilisation of numeracy practices" (p. 117) among the teachers and lead teachers surveyed when they were evaluating the 2005 Lead Teacher Initiative. These practices include numeracy activities from the resource books or website, student groupings based on strategy stage, and the use of project resources and material masters. They concluded that:

> schools appear to be developing numeracy communities of practice, with teachers involved in reflecting on their own teaching practice, collaborating with other teachers, and using student achievement information in numeracy. (Thomas & Ward, 2006, p. 117)

Looking closely at one such "numeracy community of practice" can help us to better understand the nature of sustainability for teachers and to see how the factors identified by Thomas and Ward (2006) play out in a specific school community. Country School is a small rural school that has formed its staff of six into an inquiring and focused group of teachers. Looking in depth at the experiences and practices of Country School's teachers gives us an insight into the everyday difficulties and triumphs of continuing to teach numeracy through NDP approaches.

Method

Participants

All six teachers from Country School participated in the research. Participants A and B were interviewed in 2005 and 2006. Participants E and F had returned to Country School after a year's leave. Participants C and D were new to Country School. Two teachers (A and B) agreed to be videoed. These two teachers were also videoed in 2005. The teachers' experience, class level, and facilitation history are summarised in Table 1 below.

Table 1
Summary of Participants

Participant	Years of teaching experience	Years since facilitation	Class level
A*	26	2	Yr 6–7–8
B*	24	3	Yr 5–6
C	30	3	Yr 1–2
D	7	4	Yr 6–7–8
E	3	3	Yr 2–3
F	5	1	Yr 3–4

*Video participant

Procedure

The six teachers were individually interviewed by the researcher. Each interview took approximately 20 minutes. The interviews were semi-structured, with questions about what the teachers found easy/difficult, their views on the most important aspects of the programme, and their experience of teaching in this way over several years. Interviews were audio-recorded and transcribed. Additional notes were also taken at the time of the interviews. Video recordings were made of two class lessons. These lessons were approximately 45 minutes long. The recording was done by the researcher and focused on the teacher and the children they were working with. The interviews were analysed to extract key themes and factors relating to sustaining NDP practices. The videos were analysed alongside the videos from the previous year's lessons to establish elements that had remained part of these teachers' practice.

Findings

Themes from the 2006 Interviews

The six interviews revealed commonalities across the school. Despite the fact that numeracy was not a current professional development focus, it was clearly still a matter for discussion by the staff. The teachers often used "we" to explain certain features or developments in their mathematics programmes.

> We have revised our policy at the school here to ensure we get coverage of the strands. (Teacher B)
>
> We did a pre-test because K and I work together like that. (Teacher F)
>
> That's what I find here. We can talk to each other about it as well; there are two of us at this level for starters. (Teacher D)

In describing their current mathematics programmes, the teachers all mentioned key features of NDP practice.

> I group kids by strategy stage. I have got two at the stage where they have to count every object, which would be stage 2, and then I've got a handful of stage 3, and then five stage 4, so there is quite a range. (Teacher E)
>
> The useful aspect is those books. They are fantastic and they have great ideas. (Teacher C)
>
> Having a list of the different stages you can go through once you have taught them a strategy ... having an order ... I find it on the Internet, on their website ... until they can do this, this, and this, don't go any further. (Teacher F)

All the teachers reported that there were no elements of the programme that they had consciously dismissed or dropped.

> I would carry on like this unless somebody comes up with another fantastic plan that we all have to follow. I do really like the numeracy project. I think in my classroom there is a real enthusiasm for maths and I don't think it used to be there ... Having the parents saying, "Oh, they're always talking about maths" – that's exciting too. (Teacher D)
>
> There's nothing I have changed. I've gone totally that way. (Teacher C)
>
> I took on everything because I found it fantastic. I was quite inspired. There are probably things I don't use, but just because I haven't been introduced to them. (Teacher F)
>
> No, I haven't stopped anything. I think basically I've kept it pretty routine. (Teacher B)

There were concerns expressed about planning for mathematics. This seemed to go beyond the basic act of planning – the teachers were describing the consequences of knowing what children's needs were. Their awareness of the children's strategies and knowledge had led them to devise programmes that were tailored to meet these needs. This had resulted in an increased planning and preparation burden.

> At least half an hour a day just thinking about yes, we are doing this and how am I going to teach it and what activity am I going to have to support that for that group, then the next group ... this is their activity and that game – that's going to help them support what they have learned and then the third group. I mean really, you plan six sessions at once and I find that really hard. (Teacher B)
>
> Every day you have to think: where are we at today and where are we going tomorrow, and it's just huge and I expect it to get less but it's not. You know, you think, well, what is the right one to use. There are just so many options. I mean, even the "Figure It Outs". There are so many things there to cater for the one objective and you think, well, what is the best one, and have I got the best one, and that's all your time taken. I mean, when I get down to the Figure It Outs, I just stand there for half an hour and that's it ... it's choosing the right one, the right activity and the right strategy. (Teacher A)
>
> I'm not sure whether planning is the right word either. It's just knowing where to go from here to there. (Teacher D)
>
> The planning is enormous. I find it hard to cater for everybody. I sit down half an hour before school and I have to get it ready – I can't leave it till the last minute ... I think the planning and actual work involved is massive if you want to do a good job. (Teacher E)

This concern was linked to a desire to become more fluent in their numeracy teaching. For some teachers, this was about mastering new levels they were teaching; for others, it was about having additional resources. When asked about how they hoped their mathematics teaching would look in three years' time, all the teachers said that they thought it would be very similar to their current practice but "better". Suggestions for what would constitute "better" focused on the teacher feeling more secure in their knowledge of NDP approaches and resources so that it came more easily to them and the ability to "be creative" within the programme. This creativity revolved around providing variety for the students and for themselves.

> I guess I'd like a book that has extensions of the activities, supplementary activity books published every two years ... maybe some teaching ideas about how we can do this activity or variations of because I just think kids must be bored out of their brains some days seeing the same thing every day. (Teacher B)
>
> I would like to see more books added because I think that three years down the track we are doing the same old, same old ... You think, gosh, this is getting boring, I wish I had something else ... We get resources all the time, but it's knowing what's there ... I would like to think that in time I would be more creative, but we need to keep things revitalised. (Teacher C)
>
> I think the numeracy books are fantastic, but they are quite limited – you know, you have done that and that's it, so how can I teach the same thing in a different way that's still the same? In the juniors, you can do "today we are using cars and tomorrow we'll use teddy bears" and the kids don't know they are learning the same thing, but with the seniors, you can't do that. (Teacher D)

A school-wide concern that the staff had considered together was the role of other strands in the mathematics curriculum and how these should be addressed. In 2005, the school had decided to just master the numeracy approaches and did not systematically address strands. Achievement data collected through Progressive Achievement Test (PAT) testing at the beginning of 2006 showed that the children of Country School were achieving above expectation in number but below expectation in other areas. This had led to a discussion of how to redress this, with the staff deciding to teach blocks of work on the other strands each term. This was tackled in different ways by staff in different areas of the school. Some continued with evidence-based grouping, while others went to a whole-class format. This seemed to be related to whether they saw the mathematics of the strand work as linked to numeracy or not. Teacher F and teacher B express this contrast:

> We just stopped and did a four-week block on measurement, and I find now I have to get back into the numeracy project all of a sudden. They don't blend in. They could blend in because you know measurement is all counting, doing things like that, but it's quite separate. (Teacher F)
>
> I think strand teaching is basically teaching vocab ... I did a test on children, they had to measure the perimeter of something. Because they didn't know the [word] perimeter, they got it wrong. Teach them the word "perimeter" and they can add the numbers together or multiply them, so they still have to have their number knowledge. I think it's just getting them to transfer what they know into aspects of their daily lives. (Teacher B)

All of the teachers mentioned the importance of external input into their practice. They had all appreciated the role of the facilitator in their classes, and although they had an active lead teacher, they expressed a desire for ongoing input into their mathematics teaching. The junior part of the school had employed a consultant in 2006 to come and share some ideas with them. There was a call for an informal "question and answer" and sharing time between colleagues, where teachers could feel they had been updated with changes to resources or approaches. Two desires seemed to drive this – firstly, to be "revitalised" and secondly, to make sure they were "doing it right". They found keeping up with their daily programmes and keeping abreast of changes and developments challenging.

In order to understand how these feelings and actions have changed over time, two case-study descriptions are presented. Teacher A and Teacher B both gave interviews and allowed lessons to be videoed in 2005 and 2006. This longitudinal data permits consideration of the elements of practice that have been sustained over this two-year period.

Teacher A: 2005 and 2006

Teacher A described a revolutionary shift in practice in 2005. He was inspired by the results of testing his students to engage fully with the programme approaches. He also attributed his continuing with NDP practices to the lead teacher's enthusiasm and the push for the approach from other staff.

> And the assumptions you make about some kids – and you are wrong, you know, and that was interesting. That was the really strong thing, the real analysing of the way kids are thinking and the stages ... It was just so exciting and it was a big shift, I have to say, for me personally. I had two teachers pushing me too – they were very vocal about it ... If I didn't have the push from the bottom of the school and a push from L, I may have gone back. (Teacher A, 2005)

As Principal in 2005, he had made a decision to allow classroom practice to "bed in" before altering school policies. By 2006, new policy was in place and he reported that the staff felt there was alignment between the policy and their practices.

In 2005, his only concern with the programme was preparing his year 8 students for secondary school, particularly in the other strands of the mathematics curriculum.

> I would hate to feel that they went to college not knowing as much as they did before. I am more relaxed about it this year than I was last year, sending these kids to high school. I will do this until term 3, and then term 4, teach them what they need. (Teacher A, 2005)

In 2006, Teacher A reported continuing to use the NDP approaches with his class. His concerns about teaching other strands had been confirmed by PAT data, and he had sought to add more of this to his programme. He remained enthusiastic and committed to the approach because he believed it was producing excellent results for children.

Teacher: 2005 and 2006

Teacher B's 2005 interview responses focused on the process of implementing the NDP approaches in her classroom. When asked to discuss the most useful part of the NDP, she responded:

> That would be the resource book, it's very good ... just the process of teach, follow up, and activity again, those three and making those go round. Before I did work, teach, work, teach ... Now I think it has more purpose. (Teacher B, 2005)

Her feelings about the most difficult part of the NDP also reflected organisational practices.

> When they have done their mat session, when they have done their activities that I want them to do, then they have a game ... I try and be quite specific with what I want them to do to ensure they get the most out of the game. That's the hardest thing ... to make them get the objective and be learning when they're away from you. (Teacher B, 2005)

In terms of sustaining what she was doing, Teacher B had concerns about resources.

> When you get a child at stage 6 at year 5, which I have, where do I go for resources? (Teacher B, 2005)

In 2006, Teacher B felt that she had moved to something that fitted in with her teaching rather than trying to master an external structure.

> Being off the contract, the contract said A, B, C, do this, this, and this, and now we are off the contract, I have found something that suits me – still working within the philosophy of the numeracy project, but something that fits my classroom teaching. There is more flexibility within the grouping, I'm more inclined to move children between groups and not work three groups. Sometimes I use the whole class, and sometimes I work with two groups depending on the needs. (Teacher B, 2006)

This shift away from adhering to a perceived formula had made her more comfortable with her teaching and less concerned about organisational aspects of the mathematics lesson. Her responses in 2006 focused more on student learning and on the nature of her interaction with the children.

> I think a lot of the peer sharing. I find that quite good because that makes every child think and it is expected that they will give an answer. Children ask many questions and I answer them with a question back rather than me giving them an answer, whereas I think probably two years ago I would have given them the answer. (Teacher B, 2006)

In 2005, Teacher B used the NDP resource books thoroughly and carefully and felt reliant on them. In 2006, she reported less reliance on the books and more confidence with the approach.

> I still use it as a guide, and yes I am still looking back at it, I try to head to those activities, so yes I still use them. (Teacher B, 2006)

Video lessons 2005 and 2006

The lessons that were videoed in 2005 showed that Teacher A and Teacher B both used organisational patterns and materials that could readily be associated with NDP approaches. These lesson features, such as using the resource books, using the recommended equipment and materials, grouping the class by strategy stage, sharing learning intentions, and using modelling books to record, were observed in both lessons. While each teacher had a preferred way to structure group discussion, both used peer-sharing before feeding into larger group discussion, seeking reasoning, and expecting explanations. During the sharing of strategies in group work, the interaction was between the teacher and the students in turn. Apart from one-to-one sharing in pairs, which was not heard by the group altogether, there was little discussion among the students.

The 2006-videoed lessons provided interesting insight into the effects of the NDP on practice in mathematics across the strands. As noted above, a key concern for Country School was the achievement data for their students in strands other than number. The lessons observed in 2006 reflected this because they were measurement lessons. However, these lessons included both the superficial features and interaction patterns of the numeracy lessons seen in 2005. Both Teacher A and Teacher B had transferred elements of their number teaching to the measurement material. They sought strategies from the students, using measurement as a context to consider number concepts. The students were grouped on the basis of evidence about their knowledge of measurement and their strategy stage. In the interviews, the teachers described how their approach to teaching the strand material had changed since undertaking NDP facilitation.

> It has changed what I do. I am more focused on little bits and steps, not trying to get full coverage. I think I relate it back all the time, and they are finding they are getting back to multiplication again. I think it's bringing numeracy back into our strands. (Teacher B, 2006)

> I am sort of in the habit now ... We wouldn't have grouped them like this in the past, we would have grouped them at the beginning of the year. It's all very flexible, the task books and the baskets we carry over. (Teacher A, 2006)

Teacher A and Teacher B appear to have internalised some principles of NDP approaches and have generalised these to teaching "non-project" material. Elements of this way of approaching mathematics teaching and learning appeared to have become "second nature".

Discussion

This case study is necessarily limited. It tells the story of sustained practice within a particular context. Findings within this context may or may not be applicable to other schools and other locations. The particular features of this school – its size, location, supportive community, strong leadership, and collegiality – support the practice of the classroom teachers in ways that other schools may not be able to. The willingness of the school to participate for a second year implies that the staff is eager to engage and to talk about practice.

The data does allow us to consider three key themes that add to the picture of sustainability provided by the larger-scale quantitative studies (Thomas, Ward, & Tagg, 2005; Thomas & Ward, 2006).

Firstly, schools that have been focusing on the number aspects of numeracy as they grapple with implementing the NDP may then turn their attention to the role of the other strands in the curriculum. The introduction of the new curriculum may also impact on this as teachers try to work out what is and is not in the new document and try to teach from it. In the case of Teachers A and B, the techniques, interaction patterns, and organisational approaches they had learned through the NDP had been adapted to work with objectives in measurement. This transfer suggests the internalisation of principles that could guide effective practice across the curriculum.

Secondly, the teachers articulated the effects of knowing what the children need and how that affects planning for learning. The issues raised about planning focused not so much on the act of planning as on the thinking required to meet needs and to provide an adequate programme. The teachers were concerned about choosing the right activities, making sure children were engaged with worthwhile materials, and targeting instruction for "where to next". This had placed an increased burden on them in terms of preparation, but they had not taken short cuts. The recognition of the children's needs was leading to more carefully planned instruction.

Thirdly, this case study casts some light on the role of the resource books in sustaining NDP practices. The teachers value the books highly and want more books and resources to alleviate perceived boredom and to add variety. The extent to which "the books" are seen to equal "the project" can be seen in some of the teachers' responses, where, when asked to discuss the NDP, they discuss the books (for example, Teacher B in 2005). However, it might be considered that sustained practice has been achieved when the books play less of a role in practice that is driven by a deep understanding of the children's needs and the mathematics to be taught. Teacher B in 2006 illustrates the beginnings of this shift. From a professed reliance on the books in 2005, she now feels they are more of a guide from which she selects and adapts activities. To the extent that she is able to do this in line with NDP goals, it signals the development of increasingly internalised practice.

The NDP has had a profound effect on Country School. The impetus provided by the facilitation has led to ongoing engagement with issues in numeracy learning and a commitment by the whole staff to pursue NDP teaching approaches. The benefits of this can be seen by the staff and community in improved performance in number on measures such as the mathematics PAT. Present in the school are the three factors noted by Higgins (2004): personal internalisation, collegial support, and school-wide commitment. Within this community of practice, the issues of strand coverage, planning, and resources are challenges to be discussed and worked through rather than reasons for abandoning the NDP programme.

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The Numeracy Development Projects: a Successful Policy–Research–Practice Collaboration

Brian Annan
Ministry of Education
<brian.annan@minedu.govt.nz>

This paper reveals a policy–research–practice collaboration operating in the Numeracy Development Projects (NDP) that shows promise of making an important contribution to New Zealand's schooling improvement work programme. A framework for analysing collaborations within schooling improvement initiatives is used to describe and explain the sorts of learning connections that are occurring between several groups involved in the NDP. Two tiers of collaboration are apparent: strategic collaboration among policy developers, researchers, developers, publishers, and school leaders to design and evaluate the NDP; and operational collaboration among facilitators and practitioners to implement the NDP in classrooms. An important outcome of the two-tiered collaboration is practitioners buying into the use of a set of inquiry practices that check what students know and think at the beginning and end of teaching cycles. Advantages of the approach as a lever for schooling improvement centre on role clarity and what is known about effective partnerships. Cautionary comment is also provided about the slow pace of developing policy–research–practice collaborations of this nature. The development of collegial accountability is offered as a useful trade-off for a long-term sustainable solution. Concluding remarks bring together an argument woven into the entire paper that a stronger evidence base of student achievement information will help ensure that collegial accountability is as critically challenging as it needs to be to help solve the underachievement problem among New Zealand's disadvantaged students.

Introduction

A recent study found that the Numeracy Development Projects (NDP) are showing promise as an effective schooling improvement initiative (Annan, 2006). A schooling improvement initiative is defined in that study as a planned intervention designed to raise overall academic achievement of targeted students. That definition, derived from Gray, Hopkins, Reynolds, Wilcox, Farrell, & Jesson (1999), treats raising academic achievement as the primary purpose of schooling improvement. A priority group of students in New Zealand targeted for involvement in schooling improvement are those underperforming within economically disadvantaged communities, many of whom are Māori (New Zealand's indigenous people) and Pasifika in origin (Alton-Lee, 2005). Interventions that tend to get labelled "schooling improvement initiatives" in New Zealand are typically those developed by small groups of schools working together in geographic clusters with local officials to solve common problems. One senior Ministry official likened the approach to helping schools customise their own solutions locally in much the same way that cottage industries operate (L. Whitney, personal communication, November 28, 2001). Since the mid-1990s, the schooling improvement division of the Ministry of Education has worked towards sponsoring about 20 cottage industry-style initiatives at any one time, involving approximately 10% of schools in New Zealand (Ministry of Education, 2007; Sinclair, 1999).

In recent years, the notion of schooling improvement has broadened to include a more diverse range of interventions. For instance, some national professional development strategies, including the NDP and the Literacy Professional Development Programme, are getting involved in this important work (Annan, 2006). So there is more of a national–local feel to schooling improvement. The important point here is that a combined national and local effort is more likely to impact positively on the disadvantaged student population than localised efforts alone. To take that point a little further, it would promote the creation of a comprehensive solution for the underachievement problem among disadvantaged students if national strategies containing an element of schooling improvement in them become recognised as having dual functions. For instance, the primary function of the NDP is to provide professional development for all teachers in order to raise student achievement in mathematics (Loveridge, 2003), initially through improving teachers' thinking about and understanding of number. The secondary function, how the NDP can impact more profoundly on disadvantaged students' mathematical thinking, has come to the fore in recent years as the NDP have evolved (Irwin & Irwin, 2005).

The extent to which the local and national strategies can claim themselves to be effective in terms of schooling improvement depends on the evidence they develop to show that they have successfully impacted on disadvantaged students' academic achievements. A robust way to develop strong evidence in this regard is to produce outcomes-focused studies that have three characteristics (Annan, 2006). The first characteristic is the calculation of positive and statistically significant gains in student achievement, the second characteristic is large sample sizes, and the third is replicating the findings in ten or more settings (five of which need to be comparison or third-party studies). Those three characteristics, which were found in meta-analysis methodology for schooling improvement overseas (Borman, Hewes, Overman, & Brown, 2003), provide a high degree of confidence in making claims of success about any given initiative.

The NDP are well placed to produce strong evidence of effectiveness for their schooling improvement function. They already have a well-established and accepted system for gathering and analysing achievement information. Diagnostic survey information from the Numeracy Project Assessment tool (NumPA) is already routinely collected, analysed, and used by teachers, school leaders, and national leaders to adjust lessons, school systems, and NDP developments. There is no reason why the existing achievement management system could not broaden to develop the sort of evidence that was outlined in the previous paragraph. That would simply require agreement to triangulate the diagnostic information with norm-referenced tools such as Assessment Tools for Teaching and Learning (asTTle) and Progress and Achievement Tests. The NDP trialled this approach in 2004 with a group of 16 schools serving one community, Manurewa, which has a disproportionate number of disadvantaged students. Initial results of the trial indicate that those schools appear to be achieving better results than similar schools elsewhere in the country (Young-Loveridge, 2005). It may be that a greater intensity of analysis surrounding the triangulation exercise in those schools could account for some of the success in that district.

Interesting as the development of strong evidence of the effectiveness of New Zealand's schooling improvement movement is, this article is not about that. It is sufficient to state that the NDP is well placed to develop such evidence. This paper centres on an aspect of the NDP that is another important priority for schooling improvement endeavour in New Zealand: the learning system that has been developed so that participants in the NDP learn and use effective reform practices. An analysis of the learning system attached to the NDP reveals some interesting development and implementation characteristics that participants in the NDP may find interesting for making their learning system explicit and for reflecting on its effectiveness. The characteristics may also be of interest to project leaders of other initiatives who may be reflecting on how to help participants learn what and how to do things successfully.

Fundamental to the NDP's learning system is the existence of a policy–research–practice collaboration. That means policy developers, researchers, developers, and lead practitioners work together in a non-hierarchical manner to design, implement, and evaluate the NDP. The remainder of this paper expands on this form of collaboration in three parts. The first part describes the methodology for analysing the collaboration in the NDP, the second part describes the collaboration, and the third part highlights some of its strengths and limitations as a tool for advancing schooling improvement in New Zealand.

Method of Analysis

A conceptual framework developed by Stein and Coburn (2005) to explain research–practice collaborations was adapted to analyse the NDP (Figure 1). The framework is best read by following the three trajectories from the bottom to the top. The research and development community, the policy community, and the practice community are placed alongside one another with semi-autonomous trajectories. Each trajectory begins with past understandings and practices that feed into the present improvement tasks and ends with changes being made to the original understandings and practices. They are only semi-autonomous because there is interaction between the three trajectories that cause them to learn from one another.

The ovals in the middle section of the diagram represent five working spaces in which learning can occur among those involved in schooling improvement initiatives. The first space is the school community. It is the first space because that is the space where the students learn, and what happens for them is of utmost importance. The second and third spaces are allocated to the research and development community and the policy community respectively. The fourth space is a space in which policy developers, researchers/developers, and lead practitioners collaborate to develop strategic aspects of the reforms. The positioning of the policy community below the other two groups reflects an explicit policy intention in New Zealand to promote leadership of school improvement initiatives within the research, development, and practice communities but not within the policy community (Ministry of Education, 2003). The preference is for policy to influence practitioners as an underlying support mechanism rather than to direct them through rule-governed policies from above.

The fifth space is a research–practice collaboration whereby the research and development community interacts with the practice community. Fifth space research–practice collaborations break the mould of handover encounters whereby practitioners are the recipients of researchers' findings and are left to work things out for themselves (Stein & Coburn, 2005). Instead, practitioners are reform co-constructors alongside researchers and/or developers. Initial connections between the two groups tend to help practitioners make sense of knowledge transfers to the point that they can make a start. Ongoing interactions help them acquire a deeper knowledge and sort out implementation problems as they arise. Similarly, ongoing connections help researchers understand practitioners' successes and difficulties in trying to make changes in their classrooms. Researchers/developers are, in turn, challenged to help the practitioners deal with the difficulties and achieve further successes. Therefore, critical to the formation and development of this particular sort of collaboration is the "regular, ongoing and practicable" connections between researchers/developers and practitioners (Stein & Coburn, 2005, p. 8).

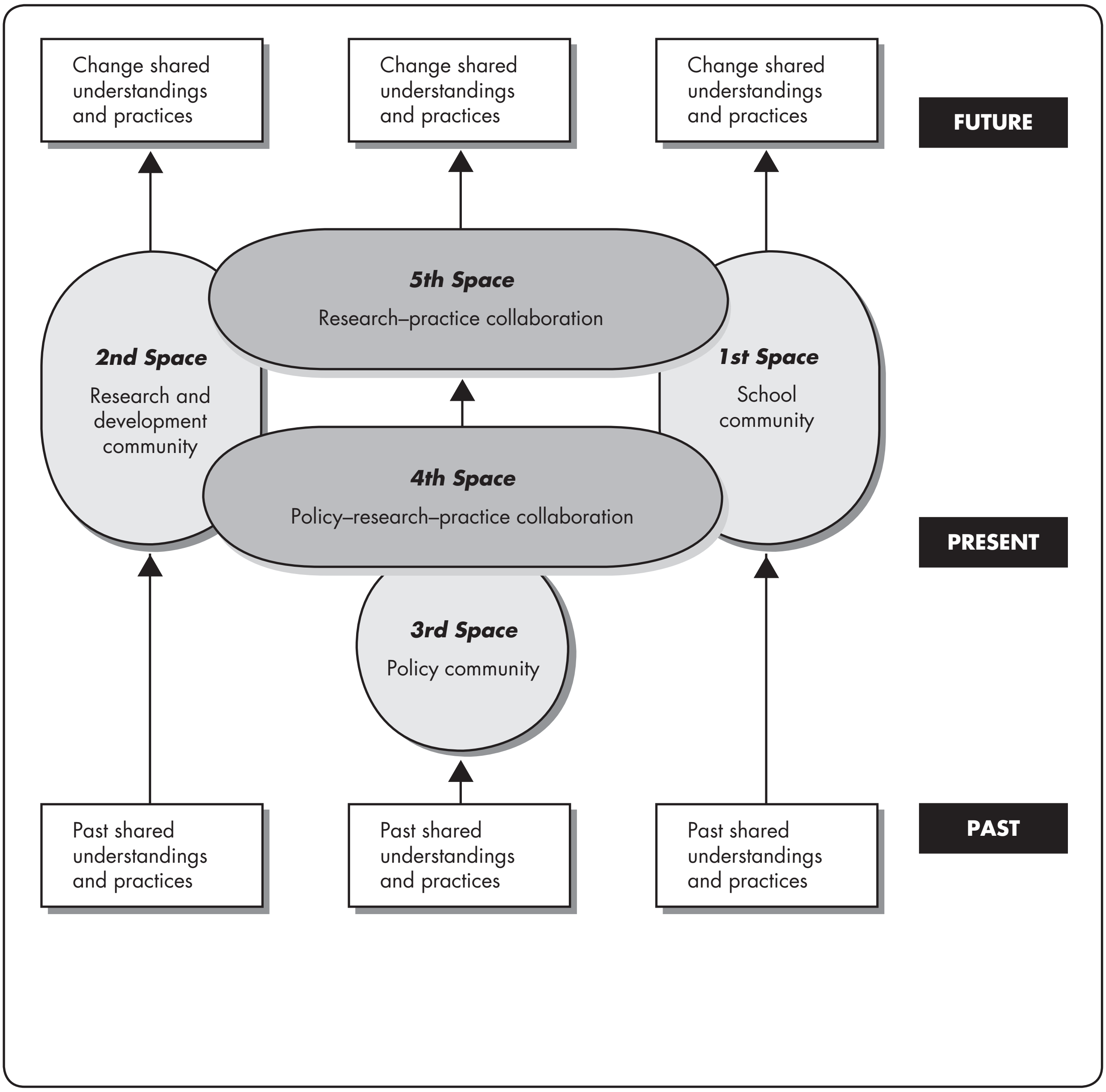

Figure 1: A framework for analysing policy–research–practice collaborations (Annan, 2006)

A Description of the Policy–Research–Practice Collaboration

The policy–research–practice collaboration attached to the NDP is presented in the diagram in Figure 2. The diagram is best read from bottom to top through the trajectories of the three groups listed at the bottom of Figure 2. The main participants fell into three main groups: the facilitators on the left, the collaboration of policy developers, researchers, resource developers, and publishers in the middle, and the teachers on the right. Information about the various groups and about the nature of their interactions with one another is organised into past, present, and future phases. In boxes at the bottom of the diagram is a description of their pre-intervention development needs from the past. In the middle of the diagram, there are a series of ovals that show the nature of the learning connections among various groups. Spaces 1, 2, and 3 represent the three groups and the set of reform practices that they used in the learning process. Spaces 4 and 5 represent the collaborations between the three groups to achieve the post-intervention understandings in the three future-focused boxes at the top of Figure 2.

Pre-intervention development needs

A key researcher contracted to inform the development and implementation of the NDP claimed that all three groups came into the NDP with specific development needs (Higgins, 2001). Facilitators tended to bring with them a tradition of putting teachers through generic professional development programmes at particular stages of their careers. They needed to shift the locus of control from themselves to the teachers so that the teachers developed an interest in understanding theories and practices in teaching number (Hughes & Peterson, 2003). The argument was that because most teachers had not been interested in theory-practice relationships, they had become stuck on written algorithms as the only way to teach students how to solve number problems. A major problem with that situation was that it was restricting the students' thinking about appropriate strategies for solving mathematical problems. One teacher's reflections on the over-reliance on algorithms showed how students' understanding about number was being ignored at the expense of efficient methods: "If they [the students] are going to do it the algorithm way, a lot of what we teach them is just a method and so they do not necessarily have a great understanding" (Higgins, 2001, p. 39). Alongside those two groups, the policy, research, resource, and publication communities tended to work separately in their own communities, typically called "silos" in New Zealand (Higgins, Parsons, & Hyland, 2002). Those developmental needs made it somewhat inevitable that the NDP were going to evolve as the three groups learnt from one another.

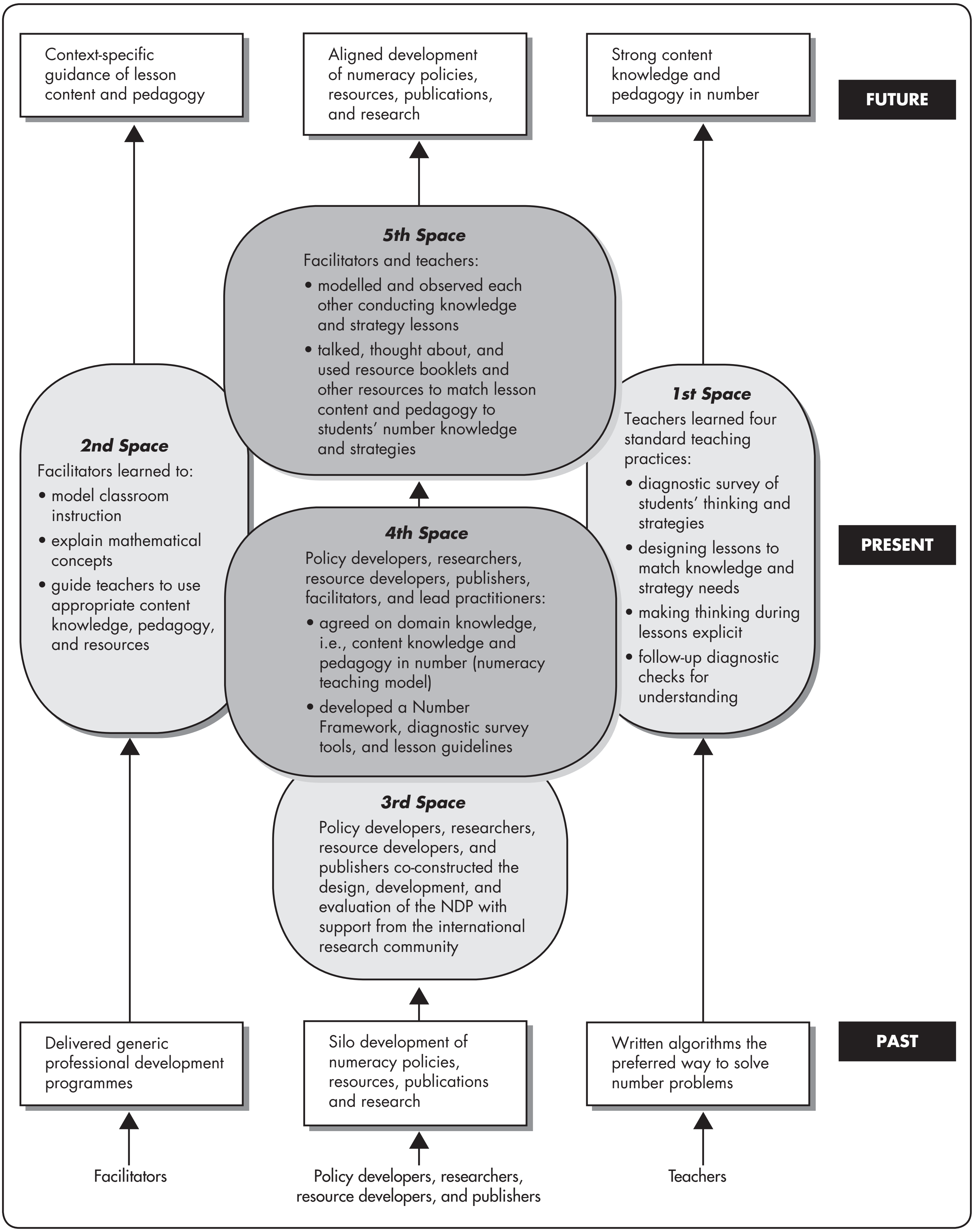

Figure 2: The policy–research–practice collaboration in the NDP

Fourth-space collaboration

The fourth-space collaboration was where important overarching reform decisions were made. An analysis of the acknowledgments in several key publications found that the membership was made up of many different communities (Ministry of Education, 2004a, 2004b, 2004c). They included representatives from the policy, research, resource publication, and international academic communities. The communities within New Zealand that became involved had some expertise to contribute to the design and development of the NDP. They also had a vested interest in seeing the NDP help improve New Zealand's mathematics ratings against other OECD countries. For instance, success mattered for the policy developers in terms of government making credible investments just as much as it mattered to the companies publishing the teaching resources in terms of their market credibility with schools (Higgins & Parsons, 2005; Higgins, Parsons, & Hyland, 2002).

The design and development team did not stop at collaboration between interested parties in New Zealand. That core team also developed learning connections with representatives of mathematics communities in England, the Netherlands, Australia, and the United States to help them along the way (Ministry of Education, 2004a, 2004b, 2004c). These extended connections meant that a multitude of communities were contributing to the design and development of the NDP. It was a complex arrangement in that some participants stayed in their own communities while others moved from one community to another. It was as if some could add greater value by staying in one space whereas others were of more value moving from one space to another. For instance, Dr Joanna Higgins from Victoria University of Wellington College of Education stayed in the research community to critique and inform the publications, processes, and outcomes (Higgins, 2001, 2002; Higgins, Bonne, & Fraser, 2004; Higgins, Irwin, Thomas, Trinick, & Young-Loveridge, 2005; Higgins & Parsons, 2005). At the same time, developers such as Peter Hughes from The University of Auckland seemed to move among various activities. He made important content contributions to the Number Framework, the diagnostic interview tools, and the resource booklets (Ministry of Education, 2004a, 2004b, 2004c). He also produced important background information for the professional development programme and helped train the facilitators.

Meanwhile two officials from the Ministry attached to the NDP worked to make sure that the various activities in the fourth and fifth space collaborations had a common and consistent policy foundation. Four foundation policy principles were outlined in a conference paper written by one of the researchers and one of the officials involved in the project (Higgins & Parsons, 2005). The writers were reflecting on the way the leaders of the NDP had gone about transferring ownership of the project from the centre to practitioners. The first three principles focus on pedagogy and teacher learning, while the fourth one centres on systemic connectivity. The principles are: (i) increasing the sophistication of mathematical ideas and teaching and learning strategies among teachers and students; (ii) ensuring that teaching decisions are informed by evidence of students' thinking; (iii) constructing teacher knowledge in their own context of practice; and (iv) adhering to a "participatory dynamic that arises from the collective agency of a complex network of overlapping groups participating in the project" (Higgins & Parsons, 2005, p. 6).

It is that latter principle that underpins the fluid movement and interactions among the participants in the strategic fourth-space collaboration operating in the NDP. Higgins and Parsons (2005) expand on the nature of the network:

> The key mechanism for the development of the collaborative process is a network of nationally coordinated groups that include a range of expertise drawn from the mathematics education community. Some of them have been established for specific purposes, that is, they are "fit for purpose", and their brief has been to address specific policy formulation, implementation or evaluation/research issues involving the schemas and resources of the project. Other groups are

> ongoing and their primary focus is aspects of the evolving design of the project. For example, the development and refinement of the explanatory conceptual framework, the diagnostic interview and the teaching model have occurred over the course of the project, incorporating new research and feedback from teachers and facilitators. (Higgins & Parsons, 2005, p. 8)

Further explanation in those conference proceedings of the network and an attached appendix explaining the make-up and purpose of the various groups (Higgins & Parsons, 2005, p. 12 of Appendix A) did not indicate that fluid movement within the network was consciously planned so that some members operated across various communities and others stayed in one. Higgins and Parsons (2005) appeared to be writing about the phenomenon after the fact. The extent to which fluidity within networks such as this one can be planned rather than just letting learning connections evolve is worthy of further investigation. It may be that planning for fluidity constrains participants' spontaneous urges to connect at the moment when they most need help to solve a presenting problem. On the other hand, it may make explicit the value of fluid movement within networks and encourage more learning connections to occur.

Among the numerous design and development decisions that the members of the fourth-space community of practice made, two were particularly important if the practitioners were to learn effective reform practices. One was deciding on the priority domain knowledge that needed to be spread among practitioners. They agreed that it should be content knowledge and pedagogical knowledge (Higgins, 2001). The second was to develop a framework, a set of standardised practices, and a set of tools to help the learning process in the fifth-space collaboration and, more importantly, to help practitioners work more effectively in their classrooms (Ministry of Education, 2004a, 2004b, 2004c). They were important decisions above others because they created a sharply-focused intervention as opposed to a general one. They also ensured that practitioners had supports for planning, implementing, and evaluating their instructional effort (National senior advisor for numeracy, New Zealand Policy Developers' Feedback, 2005). Those supports included the use of common assessment tools across the schools and the storage of aggregated achievement information (from the diagnostic surveys) in electronic databases in consistent ways for analysis and reporting purposes.

Attempting to achieve consistency in assessment procedures across schools was a bold move made by the design and development team. They were encouraging schools to relinquish their own assessment tools in favour of common ones, which could have been interpreted as an attempt to set up unhelpful external accountabilities. Feedback from one policy developer attached to the NDP indicated that, apart from a little dissent among some principals from intermediate schools about that agenda in relation to year 8 and 9 results, opposition did not happen (national senior advisor for numeracy, New Zealand Policy Developers' Feedback, 2005). A probable reason for the minimal dissent was because the assessment tools are diagnostic in nature, with the explicit purpose of helping teachers better understand the students' numerical thinking (Ministry of Education, 2004b). With that purpose being the primary driver of the initiative, teachers seemed to feel comfortable with NDP leaders external to their schools aggregating the diagnostic information and reporting effect-size gains for students' positive movements through the Number Framework stages. This situation reinforces the centrality of formative assessment as a lever for improvement in the minds of New Zealand educators.

Fifth-space collaboration

Those involved in the fifth-space collaboration put into action the strategic decisions made by the leaders of the NDP in the fourth space. Membership of the fifth space was more straightforward than the fourth space in that it involved learning connections between professional development facilitators and teachers. The facilitators had to move on from their generic traditions and work with teachers on context-specific problem solving to change their thinking. Of particular importance was the need to support teachers to be more considerate of options beyond written algorithms to solve mathematical problems. Learning connections to do so centred on observing, modelling, talking, and thinking about the content and pedagogy used in mathematics lessons (Ministry of Education, 2004b). The approach relates to the third policy principle mentioned in the previous section, that is, constructing teacher knowledge in their own context of practice (Higgins & Parsons, 2005). Hence the connections occurred in and around classrooms. There was no one specific learning place, such as the staffroom or an off-site professional development seminar room, where practitioner learning is often assumed to be occurring. It was more a process of learning on the job.

Feedback from one policy developer attached to the NDP stated that the process was challenging, particularly at the outset of the professional development programme (national senior advisor for numeracy, New Zealand Policy Developers' Feedback, 2005). The facilitators had to confront teachers' prior beliefs and practices that were not consistent with the principles underpinning the Number Framework and diagnostic interview. Those principles centred on embedding into the minds of teachers four standard teaching practices that they should use back in their classrooms once they left the fifth space:

(i) using a diagnostic survey to find out what the students know and what strategies they use to solve number problems;

(ii) designing lessons that will address students' knowledge gaps and increase their repertoire of problem-solving strategies;

(iii) teaching the lessons in such a way that the teachers' and students' thinking is made explicit;

(iv) checking that the lessons did what they were designed to do by using follow-up diagnostic surveys.

Researchers found that those practices successfully changed teachers' thinking about the importance of growing understanding before teaching algorithms (Higgins, 2001; Higgins, Bonne, & Fraser, 2004). The change in thinking was well captured in teacher interviews conducted as part of evaluations for the initial trial of the NDP and once it was well underway. One quote, in particular, captures the shift in teachers' thinking: "I think, for little kids, it's probably really important not to introduce the algorithms until you're sure they've really got it because it's an easy way of working the answer out without actually understanding how the heck you got that answer" (Higgins, 2001). So the mind shift was more about understanding the process of solving problems than about inducting students into easy ways of producing the right response.

Discussion

A useful catchphrase for the four practices developed through the NDP is "developing evidence-informed collaborative inquiry". This catchphrase is the title of a book chapter that explains the four practices and the theory underpinning them in several other schooling improvement initiatives (Timperley, Annan, & Robinson, in press). The four practices are intentionally sequenced in such a way that a problem analysis is at the forefront of any inquiry to improve teaching and learning. Additionally, the sequencing ensures that an outcomes-focused evaluation concludes such inquiries. That sequencing is essential to get away from a popular New Zealand cultural norm of just getting on with the job, regardless of the outcome (Annan, 2006). That norm is typically referred to as the "No. 8 wire approach" (Hopkins & Riley, 1998). Number 8 wire is a common type of fencing wire used in New Zealand to fix things around the house or farm. A less than desirable outcome is beside the point. At least the problem is dealt with and further tinkering can be done if necessary. This garden-shed approach may be a cost-effective way of handling many everyday problems in a small isolated country with limited funding. However, it lacks the sophistication required to solve priority national educational problems such as helping a large number of mostly disadvantaged students who are unable to solve number problems and read at the level required to become successful (OECD, 2001). A more sophisticated scientific approach is better suited to solve those sorts of complex problems. Hence the sequencing of the four practices so that regular checks on student achievement trends are completed as a matter of course.

An advantage of evidence-informed collaborative inquiry is creating role clarity among groups partnering to do schooling improvement work. Partnerships often languish because they concentrate more on developing relationships in the name of collaboration than on allowing roles and relationships to evolve out of a priority task (Timperley & Robinson, 2002). In the NDP, relationships formed around the priority task of improving practitioners' and students' thinking about and understanding of number and how to use that understanding to solve number problems. Through an evolutionary development process, the various partners worked out their places in designing, implementing, and evaluating the initiative (Higgins & Parsons, 2005). The NDP ended up with two tiers of collaboration: a strategic tier that involved policy developers, researchers, developers, publishers, and lead practitioners taking responsibility to lead the design and evaluation of the NDP and a more practical tier involved facilitators and the practitioners implementing the NDP in classrooms.

Distributed responsibility and accountability of the nature established across the various groups involved in the NDP fits with a useful definition of partnership for schooling improvement put forward by Timperley and Robinson (2002): "We propose that entities are in partnership when they each accept some responsibility for a problem, issue or task and establish processes for accomplishing the task that promote learning, mutual accountability and shared power over relevant decisions" (p 15). What is really heartening about the partnerships formed through the NDP is that they have touched almost every teacher and student in the country (Higgins, Parsons, & Hyland, 2002). That is not to say that localised endeavours are not worthwhile. To the contrary, localised efforts often capture important context elements that national strategies can easily overlook (Kliebard, 2002). In this regard, the work that the NDP did with the local leaders in the Manurewa district to roll out the professional development and to triangulate their successes offers a way of getting the best of both worlds. That national–local collaboration shows how a local infrastructure can help contextualise a national solution.

A caveat for the way evidence-informed inquiry is developing through New Zealand's various schooling improvement efforts is that an evolutionary process for working things out through partnerships without too much rule-governed supervision is a slow process. It has taken the best part of a decade to prioritise the inquiry practices outlined in this paper. The process of spreading them among all those that need to use them across the curriculum is largely work yet to do. What that long game means is that the NDP has done a good job of spreading them in numeracy and the Literacy Professional Development Project, coupled with numerous cottage industry initiatives, is starting to get a good spread in literacy. However, there is no guarantee that all practitioners will continue to use them as designed or will transfer them to other areas of the curriculum. In other words, programme integrity is discretionary (Annan, 2006). So even if the practices are spread more widely, practitioners can choose to go back to personally preferred but less effective practices. This is where collegial accountability is so important for the long-game approach to schooling improvement to be successful. It is imperative that practitioners check on each other for slippage from the effective practices established through an initiative.

Conclusion

Policy-research-practice collaboration in the NDP is successfully spreading evidence-informed inquiry practices that have proven to be central to effective schooling improvement. Promoting those practices is helping create the sort of critically challenging schooling culture necessary to solve complex underachievement problems. Accelerating a school culture change of this nature is a fairly urgent and important policy challenge if the underachievement problems are to be solved in the foreseeable future. One accelerant proposed in this paper is to help practitioners hold themselves and each other to account for the way they are performing in their various roles. Developing collegial accountability over external accountability helps situate the ownership of the problems among those most closely associated with having to solve them, that is, those working in and around classrooms. Important context-specific assessment work is under way in the NDP that will support practitioners to hold each other to account. Important triangulation exercises are underway to check the NUMPA results with norm-referenced evaluation tools such as asTTle. Replication of those exercises for all schools will help practitioners to analyse, critique, and challenge the effectiveness of their own, each other's, and, most importantly, the students' practice.

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What Do Teachers Know About Fractions?

Jenny Ward
Maths Technology Ltd.
<jenny@nzmaths.co.nz>

Gill Thomas
Maths Technology Ltd.
<gill@nzmaths.co.nz>

This paper reports on the development and initial trialling of a tool to assess teacher knowledge of the teaching of fractions. Results showed that a pen-and-paper assessment focusing on teachers' pedagogical content knowledge can be both efficient and effective in differentiating between teachers. In general, teachers scored more highly on questions requiring a response based on their content knowledge than on questions where responses required them to describe the key ideas involved or the actions they would take with a student. Questions that caused teachers difficulty involved the addition of fractions, division with fractions, and proportional reasoning, with approximately one-third of teachers' responses indicating a lack of conceptual understanding in each of these areas. These results suggest that the knowledge of teachers may be a factor limiting student achievement, particularly in the proportions and ratios domain of the Number Framework. Further work is required to establish a link between teachers' scores in the assessment and student achievement data. If the validity of the assessment tool can be established in this way, its use to tailor teachers' professional learning may be significant.

Background

It is widely accepted that the knowledge a teacher holds affects the way they perform all the core tasks of teaching (National Council of Teachers of Mathematics, 2000). In particular, a teacher's knowledge of subject matter, student learning, and development and their teaching methods have all been identified as important elements of teacher effectiveness (Hammond & Ball, 1997). Focusing on a teacher's knowledge of content, Shulman (1986) defined pedagogical content knowledge (PCK) as knowledge of a subject "for teaching". He differentiated this from pure subject knowledge by describing PCK as including the best ways to present the subject to learners, the most useful examples to use to illustrate certain points, and an idea of the misconceptions and preconceptions that learners may bring with them to the learning.

Since Shulman's definition of PCK, researchers have worked in many subject areas to investigate teacher knowledge and map the precise knowledge a teacher requires to be effective (Hill, Schilling, & Ball, 2004). In mathematics, these investigations have included comparing the views of pre-service teachers with those of experienced teachers (Ball, 1990), in-depth interviews with practising teachers (Harel & Lim, 2004), and comparisons of the differences in teacher knowledge across cultures (Ma, 1999). In focusing on teacher knowledge in mathematics, researchers have made a distinction between teachers that have an algorithmic or rule-based understanding of mathematics and those that have a deep conceptual understanding.

Difficulty with fractions among teachers is well documented in many countries, and many authors consider fractions to be the most difficult area of mathematics covered in primary school (Smith, 2002). Studies into teacher knowledge of fractions have found both procedural and conceptual understandings among teachers, although procedural understandings dominate in this area (Fuller, 1997). Considerable differences have also been found in the explanations teachers provide to students when working with fractions. Some teachers use significant conceptual information in their explanations, while some focus more on algorithms and procedures (Leinhardt & Smith, 1985). When looking at the representations that teachers use to present fractions to students, a limited repertoire has been found (Ball, 1990). Circular representations are most commonly used, but these can be problematic because they are unable to illustrate conceptually-complex operations with fractions, such as division.

In addition to defining the knowledge required to teach mathematics effectively, recent studies have also sought to measure teacher PCK in mathematics (Hill, Schilling, & Ball, 2004). This work has been followed closely, with investigations of the relationship between student gains and teacher knowledge. These investigations have revealed a non-linear relationship between student achievement data and teacher PCK scores. In particular, the teachers in the bottom third of the knowledge distribution have a significant negative impact on their students' achievement (Hill, Rowan, & Ball, 2005). For those working in professional development, this work highlights the need to differentiate between teachers on the basis of their existing knowledge and to provide focused professional development for the third of teachers with the least knowledge.

In New Zealand, the Numeracy Development Projects (NDP) have dominated professional development initiatives in mathematics education over recent years. The NDP aim to improve students' use of mental strategies to solve number problems by focusing on the professional knowledge of teachers, and there is much evidence to suggest the projects have been effective in raising student achievement (Bobis, Clarke, Clarke, Thomas, Wright, Young-Loveridge, & Gould, 2005). Despite this increased achievement, teacher knowledge remains an area of concern. A recent report from the Education Review Office claims that 23% of teachers hold only partially effective or not effective pedagogical content knowledge in mathematics (Education Review Office, 2006). Alongside this, student achievement data from the NDP indicate that multiplicative and proportional thinking remain areas of difficulty with student performance "a little disappointing" in these domains (Young-Loveridge, 2006).

> The data indicate that there are important issues to investigate further with respect to multiplicative thinking and understanding of fractional numbers. (Young-Loveridge, 2006, p. 20)

This paper reports on work undertaken in 2006 to develop an efficient and effective tool to assess teacher knowledge of the teaching of fractions. This information could be used to tailor professional development programmes to meet the needs of individual teachers.

Method

Development of the Assessment Tool

The assessment tool was developed with the assistance of a reference group consisting of five primary teachers and a numeracy facilitator experienced in working with teachers in the NDP. The group met three times with the researcher to discuss the PCK required by teachers to be effective in teaching fractions. This group provided feedback on the form of the assessment, assisted with the drafting of questions, conducted a small-scale trial of alternative wordings for questions, and helped modify the assessment on the basis of these trial results.

The final version of the assessment tool comprised seven questions based on scenarios involving the teaching and learning of fractions. In general, questions described a scenario and then asked teachers to first identify the mathematically correct answer to the problem posed and then describe either the key understandings involved or the teacher actions required. Each question required two or three responses, depending on the nature of the content covered, with a total of 17 responses required to complete the assessment. All questions included an option for teachers to indicate if they were unsure of how to respond.

The content areas covered in the assessment were the comparison and ordering of fractions, addition of fractions, proportions, fractions greater than one, division of fractions, and equivalent fractions.

Trial of the Assessment Tool

The assessment tool was trialled in two schools selected from the 22 primary schools involved in the 2006 Longitudinal Study. The Longitudinal Study has been ongoing since 2002 and examines the impact of the NDP in schools that have been involved for a number of years (Thomas & Tagg, 2004, 2005, 2006; Thomas, Tagg, & Ward, 2003). It uses a sample of schools selected to be representative of the national sample.

One high-decile school and one low-decile school were selected to be involved in the trial of the assessment tool. Both schools were located in the North Island, and each had approximately 20 classroom teachers. Table 1 displays this information. Participating teachers were predominantly full-time classroom teachers, but part-time classroom teachers and management staff were also included.

Table 1
Participating Schools

	School one	School two
Decile	1	8
First participation in NDP	2001	2002
Number of participating teachers	21	23

One of the senior management team at each school administered the assessment, following a set of instructions provided by the researcher. The assessment was administered at staff meetings, with teachers completing it individually. Teachers were given as much time as required to complete the assessment.

A feedback questionnaire was used to gather information on participating teachers' perceptions of the assessment. The questions asked teachers to describe their feelings about the assessment and to rate and describe their own knowledge of fractions.

Analysis of Results

Teacher responses to each of the questions were grouped and used to develop marking criteria. In general, three categories of answer were identified. These were: correct answers that identified the key concepts involved and explained or described these fully; answers that showed some understanding by identifying key concepts but omitted an explanation or description of these; and incorrect answers that either identified irrelevant or unrelated concepts or were too general and broad.

The marking criteria developed were used to mark the assessments completed by the teachers, with each correct response scoring one point and a total of 17 points possible. An analysis of scores and responses was then undertaken. Where reported percentages do not add to 100, this is due to rounding error.

Findings

Usefulness and Practicality

In general, the assessment was practical to administer and produced a range of scores that could be used to differentiate teachers. The average time taken to complete the assessment was approximately 14 minutes, with the times for completion ranging between 5 and 22 minutes. Table 2 shows the spread of teacher scores. One teacher recorded correct responses for all questions and received full marks of 17 points; one teacher received a score of zero, with no correct answers. Almost every score possible was recorded by at least one teacher.

Table 2
Teacher Scores

Score	0	2	3	4	5	6	7	8	9	10	11	12	13	14	17
Number of teachers	1	1	1	2	6	5	2	7	2	3	3	5	4	1	1

Responses in the feedback questionnaire showed that most of the teachers involved did not find the assessment stressful. Thirty-six percent of teachers were neutral or relaxed about being asked to do the assessment, while another 36% reported feeling slightly anxious. Ninety-three percent of the teachers reported either being more relaxed after the assessment or no change in their level of anxiety before and after the test, and just 5% of the teachers felt more anxious after the assessment. In general, teacher comments reflected an open approach to completing the assessment.

> I think I've done ok!
>
> I know more than I thought I did.
>
> Glad I could do it (I hope I got it right.)

Teacher Knowledge of Fractions

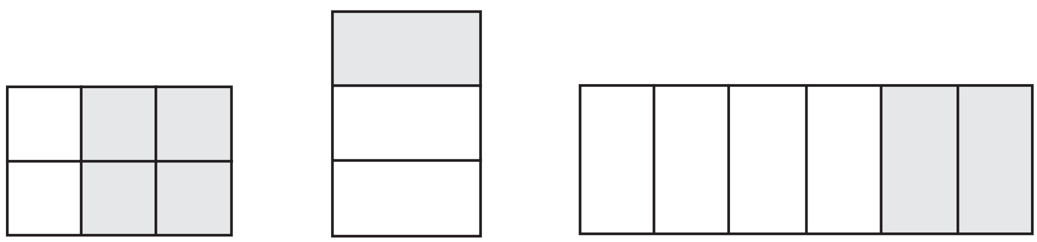
Table 3 shows participating teachers' scores in each of the content areas covered. As described previously, the first part of each question explored the teachers' own knowledge of fractions (noted in the table by shaded rows) and the later parts probed teachers' conceptual understanding and their ability to communicate this understanding to students – their PCK. Questions had either two or three parts, depending on the nature of the scenario.

Table 3
Teacher Scores by Question Content
Shaded rows indicate content-only questions; non-shaded rows indicate PCK questions.

	Responses			
	Correct % (n)	Show some understanding % (n)	Incorrect % (n)	No response % (n)
	59 (26)	na	36 (16)	5 (2)
1. Comparison and ordering of fractions	55 (24)	na	36 (16)	9 (4)
	32 (14)	9 (4)	11 (5)	48 (21)
2. Addition of fractions	66 (29)	na	14 (6)	20 (9)
	9 (4)	11 (5)	66 (29)	14 (6)
	68 (30)	na	27 (12)	5 (2)
3. Proportional reasoning	43 (19)	9 (4)	11 (5)	36 (16)
	50 (22)	9 (4)	11 (5)	30 (13)
4. Improper fractions	84 (37)	na	2 (1)	14 (6)
	45 (20)	34 (15)	7 (3)	14 (6)
	48 (21)	na	39 (17)	14 (6)
5. Division involving fractions	59 (26)	5 (2)	14 (6)	23 (10)
	18 (8)	9 (4)	36 (16)	36 (16)
6. Equivalent fractions	89 (39)	na	9 (4)	2 (1)
	18 (8)	39 (17)	27 (12)	16 (7)
7. Ordering unit fractions	86 (38)	na	2 (1)	11 (5)
	48 (21)	20 (9)	20 (9)	11 (5)

In general, teachers scored more highly on questions requiring a response based on their content knowledge than on questions where responses required a description of the key ideas involved or the actions they would take with a student. On questions involving content, the average percentage of teachers recording correct responses was 69%, while on questions involving PCK, the average percentage of teachers recording correct responses was 36%. This trend is illustrated by teacher responses to question 6, which assessed teacher knowledge of equivalent fractions (see Figure 1).

Which shape has $\frac{2}{3}$ of its area shaded?



Mark insists that none of the shapes have $\frac{2}{3}$ of their area shaded.

Do any of the shapes have $\frac{2}{3}$ of their area shaded?
What action, if any, do you take?

Figure 1: Question 6, equivalent fractions

Eighty-nine percent of teachers correctly identified that one of the shapes had two-thirds of its area shaded. However, only 18% of teachers clearly described how to use materials to demonstrate that two-thirds is equal to four-sixths. One of these responses is illustrated in Figure 2.

Show equivalence. $\frac{4}{6} = \frac{2}{3}$. Rearrange visually to show $\frac{2}{3}$ over the top of $\frac{4}{6}$.

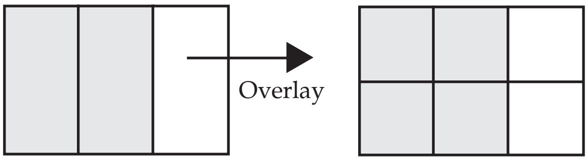


Figure 2: Teacher response giving clear description of appropriate teaching action

Thirty-nine percent of teachers were able to give a response that showed some understanding by mentioning equivalent fractions and/or the use of materials but did not clearly describe the actions they would take. Examples include:

Discuss and use materials to learn about equivalent fractions.

Equivalent fractions need to be modelled and taught.

Develop notion of equivalence.

The question for which fewest teachers answered both parts correctly involved the addition of fractions (see Figure 3).

You observe the following equation in Sally's work: $\frac{3}{5} + \frac{2}{3} = \frac{5}{8}$

Is Sally correct?

You question Sally about her understanding and she explains: I scored 3 goals out of 5 in the first half and 2 goals out of 3 in the second half. Overall, I scored 5 out of 8 goals.

What, if any, is the key understanding she needs to develop to solve this problem?

Figure 3: Question 2, addition of fractions

Sixty-six percent of teachers successfully identified the equation in Sally's work as incorrect. However, just 9% (four teachers) clearly described the key understanding she needed to develop in order to solve the problem. These responses included:

That the denominator is the total number of goals or parts for each fraction. That the denominator is the total attempts and the numerator is the number of times she succeeded. $\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$

That she is working with 8 as the whole, therefore it was $\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$

In response to the second part of the question, 11% of teachers recorded an answer that showed some understanding, while 80% of teachers were unable to identify the key understanding required, recording either an incorrect response (66%) or no response at all (14%). This was the only question for which the majority of the teachers recorded incorrect responses. Of the 29 responses categorised as incorrect, 12 teachers recorded answers that were very general and 17 teachers described the use of a common denominator to solve the problem.

> She needs to understand that you need to have a common denominator to add fractions together.
>
> She needs to change the fractions so she has the same denominator.

One teacher's response illustrated a lack of understanding of common denominators:

> Sally needs to understand about using a common denominator before adding fractions:
>
> $\frac{3}{5} + \frac{2}{3} = \frac{9}{15} + \frac{10}{15} = \frac{19}{30}$?

The use of a common denominator is a valid mathematical approach when adding three-fifths and two-thirds, but in this situation, the key understanding that Sally needs to develop is not related to the use of a common denominator. If a common denominator is used in this example, the numbers in the equation ($\frac{9}{15} + \frac{10}{15} = \frac{19}{15}$) bear no real meaning to the goals scored in the game. Sally's understanding of how to use fractions to successfully represent the goals in the two halves of the game will not be developed by developing her knowledge of common denominators. These responses indicate a rule-based or procedural approach to tasks involving the addition of fractions rather than a deep understanding of the concepts involved. The NDP seek to promote a deep understanding among teachers where:

> less emphasis is given to the rote performance of written algorithms to calculate answers. (Ministry of Education, 2006, p. 2)

A general lack of understanding of the content involved in the addition of fractions was indicated by 34% of teachers, who either stated that Sally's equation was correct (14%) or omitted to answer (20%) the first part of the question. Addition and subtraction with fractions is placed at stage 8 of the additive domain in the Number Framework. Student achievement data indicate that there are at least a small number of students operating at stages 7 and 8 from year 3 on (Young-Loveridge, 2006). Of the 15 teachers that recorded an incorrect response to this question, eight teachers identified that they teach students from these year levels. This finding may be a cause for concern because it suggests an area in which teacher knowledge may be impacting on student achievement.

Division with fractions

Another question on which teachers performed poorly involved division with fractions (see Figure 4).

> You observe the following equation in Jane's work: $1\frac{1}{2} \div \frac{1}{2} = \frac{3}{4}$
>
> Is she correct?
>
> What is the possible reasoning behind her answer?
>
> What, if any, is the key understanding she needs to develop to solve this problem?

Figure 4: Question 5, division involving fractions

Forty-eight percent of the teachers correctly identified that there was a problem with the equation in Jane's work, while 39% of teachers believed her recording was correct. Just 18% were able to clearly describe the key understanding that Jane needed to solve the problem by clarifying the conceptual question behind the equation. These responses included:

> It's asking "how many halves are there in $1\frac{1}{2}$?"
>
> That the question is asking "if $1\frac{1}{2}$ is a half, what is the whole?"

Seventy-two percent of teachers were unable to identify the key understanding that Jane required. This group included 36% who recorded no response to the question and 36% who gave an incorrect answer. Incorrect answers were very general or identified unrelated concepts. Examples include:

> Equal parts – sharing, mixed fractions, equivalent fractions
>
> $\frac{1}{2} + \frac{1}{2} = \frac{2}{2}$, denominators don't change.

Fifty-nine percent of teachers identified a plausible explanation as Jane's possible reasoning. The explanations were of two types. One was the addition of the numerators and denominators to give three over four, and the other was one and a half being divided by two instead of by a half.

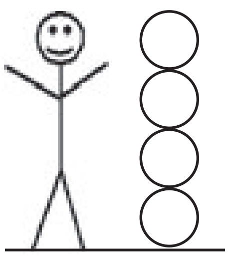
> She may have added 1 + 1 + 1 = 3 then added 2 + 2 = 4.
>
> She is dividing one and a half into 2 groups.
>
> She's just halved one and a half.

It is interesting to note that five of the 12 teachers who identified Jane's reasoning as one and a half divided in two evenly believed that her answer to the problem was correct. In general, responses to this question indicate that many of the teachers who participated lacked a conceptual framework for problems involving the division of fractions. This is consistent with other work in this area, where it has been found that, in general, teachers have a limited conception of fractions (Ball, 1990), with many unable to apply their general understanding of division into the context of fractions. Commonly, teachers regard division with fractions problems primarily as fractions problems rather than division problems. The use of language has also been found to be problematic in this area: to divide something in half is to divide it into two equal parts; to divide something by one-half is to form groups of a half. This is a distinction that teachers find difficult (Fuller, 1997).

Proportional reasoning

Disappointing student performance in the multiplicative and proportional domains over recent years has been identified as an area requiring further investigation (Young-Loveridge, 2006). For this reason, it is interesting to consider teacher responses to proportional reasoning problems such as question 3 (see Figure 5).

This is Mr Short.



The height of Mr Short is 4 large buttons.
The height of Mr Tall is 6 large buttons.

When paper clips are used to measure Mr Short and Mr Tall, the height of Mr Short is 6 paper clips.

What is the height of Mr Tall in paper clips?

Jim answers 8 paper clips.
Steve answers 9 paper clips.

Who is correct?

What is the possible reasoning behind **each** of their answers?

Figure 5: Question 3, proportional reasoning

An adapted version of the Mr Tall/Mr Short problem (Khoury, 2002)

Sixty-eight percent of teachers were able to identify Steve's response of nine paper clips as the correct answer to this problem. Of the remaining 14 teachers, 10 teachers mistakenly identified Jim's answer as correct, two teachers believed both Jim and Steve's answers could be considered correct, and two teachers did not record an answer. In total, nearly 32% of teachers were unable to answer this question correctly.

When asked to describe the students' thinking, 43% of teachers were able to describe Jim's additive reasoning clearly:

> Jim has simply retained the difference of 2 between each height.
>
> Buttons + 2 = paper clips
>
> Jim thinks there will be 2 paper clips difference, as with the buttons.

Fifty percent of teachers were able to describe Steve's proportional reasoning by describing the proportional gain or the difference in proportion between the two heights:

> Steve looked at the percentage by which each increased, Mr Short = 4 x 150% = 6, therefore Mr Tall = 6 x 150% = 9.
>
> Steve has recognised an increase of half and subsequently added half of 6, making it 9.
>
> Ratios 2:3, 4:6, 6:9

The inability of approximately one-third of the participant teachers to correctly answer this proportional reasoning question is potentially a cause for concern. Student achievement data suggests that teachers of students in years 5 and above will have at least a small number of students who are proportional thinkers in their classrooms (Young-Loveridge, 2006). Of the 14 teachers that were unable to answer this question, five identified that they were teaching in classes with students that were year 5 or higher. These results provide evidence that the knowledge of teachers may be a factor limiting student achievement in some cases and are worthy of further investigation.

Teacher Scores by Year Levels

In the development and trial of the assessment tool, many teachers working in year 1–3 classes expressed a belief that they did not require a comprehensive knowledge of fractions because this was not an area their students worked in extensively.

> Know enough to teach junior end of the school. Would have to do a refresher course for myself if I had to teach level 3.
>
> I teach juniors and we really only cover $\frac{1}{2}$, $\frac{1}{4}$ at this stage.

The results did provide some evidence of a lower level of knowledge among teachers working in junior classrooms. The 13 teachers working solely with students in years 0–2 received an average score of 7.8 in the assessment, while the 25 teachers working with students in year 3 and above received an average score of 9.9. Six teachers did not identify the level of the students they taught or identified that the students they taught had special needs. In addition, 46% of the junior teachers identified themselves as being unsure of how to respond to at least one question, while just 16% of the remaining teachers identified themselves as being unsure of one or more responses.

While it is difficult to define the precise level of knowledge required by teachers in junior classrooms, there is some justification for teachers at this level not needing to reason proportionately because they are unlikely to be required to teach proportional reasoning. Student achievement data indicate that by year 2, approximately 4% of students are able to find a fraction of a number, order unit fractions, and use symbols for improper fractions (Young-Loveridge, 2006). While there are relatively few year 2 students working with fractions, teachers need to have a sound conceptual understanding of fractions, and operations with fractions, in order to meet the learning needs of these students.

Teachers' Perceptions of Their Own Knowledge

Participating teachers were asked to rate their own knowledge for teaching fractions using a five-point scale ranging from very weak to very strong. In general, those teachers that rated their own knowledge as weak achieved lower scores on the assessment than those who rated their own knowledge more highly. Table 4 shows these results.

Table 4
Teacher Self-assessment and Average Scores

Self-assessment	Teachers % (n)	Average score
Very weak	7 (3)	4.7
Weak	20 (9)	7.2
Moderate	15 (22)	9.4
Strong	18 (8)	10.3
Very strong	2 (1)	11.0

While these results provide some evidence that self-assessment may be of value in assessing the knowledge of teachers, the large range of scores achieved by teachers within each rating limits the value of this information. For example, within the 22 teachers who rated themselves as having a moderate knowledge for teaching fractions, scores in the assessment ranged evenly from 3 to 17 points.

Concluding Comment

Results suggest that a pen-and-paper assessment focused on teachers' PCK can be both efficient and effective in differentiating between teachers. In general, teachers scored more highly on questions requiring a response based on their content knowledge than on questions where responses required a description of the key ideas involved or the actions they would take with a student.

Questions on which teachers performed poorly involved the addition of fractions, division with fractions, and proportional reasoning. Teacher responses in these areas indicated a lack of conceptual understanding, with 39% of teachers describing the use of common denominators to add fractions in a way that did not support conceptual understandings and 36% not being able to clarify the conceptual question behind an equation involving division with fractions. Thirty-two percent of teachers were unable to answer proportional reasoning problems correctly, providing some evidence that the knowledge of teachers may be a factor limiting student achievement in some cases.

Teachers working with students in year 3 and above scored more highly than teachers working with students in year 1 and 2, and this is consistent with participants' views that teachers of younger students do not need a highly developed knowledge of fractions. However, it needs to be noted that students from year 2 on will be working with fractions and teachers at this level need to have a sound conceptual understanding of fractions, and operations with fractions, in order to meet the learning needs of these students. Teachers' perceptions of their own knowledge in fractions tended to reflect their scores in the assessment; however, the large range of scores achieved by teachers within each self-assessment rating limits the value of this information.

Further work in this area is required to establish a link between teachers' scores in the assessment and student achievement data. If it can be shown that teachers with poor scores in the assessment have correspondingly poor student achievement data, the validity of the tool will be established and its use to tailor teachers' professional learning may be significant. It may also be useful to develop similar assessments based on other areas of numeracy teaching and learning.

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Exploratory Study of Home–School Partnership: Numeracy

Jonathan Fisher
New Zealand Council for Educational Research
<jonathan.fisher@nzcer.org.nz>

Alex Neill
New Zealand Council for Educational Research
<alex.neill@nzcer.org.nz>

This article summarises the findings from an exploratory study about the Home–School Partnership for Numeracy (HSPN) pilot. This study involved interviews with lead parents and lead teachers, focus groups with parents, and observations of the community sessions in three case-study schools. This data was supplemented by surveys to all the Numeracy Development Project facilitators associated with the HSPN. Findings indicated that there was strong support for the HSPN from parents, teachers, and the facilitators. Some ideas about improving the training workshops and community sessions were also suggested. Developing a home–school partnership is argued to be of great importance, and achieving the balance of inviting, involving, and informing parents is different for each school community. Important components of a successful ongoing effective programme identified were the careful selection and retention of the lead parent, responsiveness and incorporation of parents' contributions in the community sessions and the training workshops, whole-school involvement in the programme, and flexibility to enable each school to accommodate the needs of their community.

Introduction

Home–School Partnership: Numeracy (HSPN) is part of an initiative designed to raise achievement for Pasifika and other bilingual students by enhancing family and community engagement in their children's learning. In 2006, the HSPN pilot was developed by the Ministry of Education to explore the issues around implementing and sustaining a home–school partnership programme as an ongoing initiative. The pilot involved approximately 40 primary schools in six regions, co-ordinated by 15 facilitators. The philosophy (kaupapa) underlying the HSPN is expressed by two whakataukī:

> He aha te mea nui o te ao? He tangata, he tangata, he tangata!
> What is the most important thing in the world? It is people, people, people!
>
> Nāu te rourou, nāku te rourou, ka ora te iwi.
> With your food basket and my food basket, everyone will have enough.
>
> (Ministry of Education, 2003, pp. 121–122)

The essence of the first whakataukī is the importance of all people in the programme: the parents, families, children, teachers, principal, school staff, and wider members of the community. The second whakataukī illustrates the importance of partnership – "genuine sharing, hospitality, and reaching out to others" (Ministry of Education, 2003, p. 122).

The HSPN was derived from a similar programme for literacy developed in 2003, Home–School Partnership: Literacy (HSPL). Both programmes use a similar structure and share a significant amount of support information from the HSP resource folder (Ministry of Education, 2003 for literacy and 2004 for numeracy).

The HSPL had its origins in the 2001 Pasifika Education Plan, which identified a number of important goals for the focus of "increasing achievement in early literacy and numeracy, attainment of school qualifications, and reducing at-risk factors" (Ministry of Education, 2001, p. 2). The importance of developing closer relationships between home and school was stated as one of these goals, and home–school partnerships was identified as one way "to increase and strengthen school liaison with Pacific parents and communities" (Ministry of Education, 2001, p. 2). The idea of involving parents in their children's learning is clearly stated in the Ministry of Education's statement of intent, which emphasises the importance of "family and community engagement in education" (Ministry of Education, 2006, p. 12). The Literacy Task Force (1999, p. 4) further identifies that "children's learning is enhanced by effective partnerships between school and home" and involves good communication and shared understandings.

Many aspects of the HSPN have been informed by a plethora of literature, home–school relationship initiatives, and mathematics education and are well grounded in contemporary research. Some key findings are the benefits of:

- involving parents in a two-way partnership (Alton-Lee, 2003; Biddulph, Biddulph, & Biddulph, 2003; Literacy Task Force, 1999; Anthony & Walshaw, 2007)
- catering for diversity in a genuine relationship (Alton-Lee, 2003)
- considering parents' and students' first language (Biddulph et al., 2003; Alton-Lee, 2003)
- having bilingual parents support other bilingual or ESOL parents (Mara, 1998)
- effective home–school partnership leading to improved mathematical dispositions and achievement (Anthony & Walshaw, 2007).

The mathematical content of the HSPN drew heavily from the strategies and knowledge in the Number Framework (Ministry of Education, 2002), which is well grounded in current mathematics education literature.

In 2006, the HSPN involved three or four training workshops run by the HSPN facilitators for a lead team of teachers and selected parents from the community (referred to as lead parents). These workshops included activity-based learning, whole-group and small-group discussion, opportunity for reflection, feedback, and time for planning. The activities presented were linked to the stages of the Number Framework. The facilitator, the lead team, and the school used the HSP resource folder to support the organisation and content of the workshops. Each workshop was followed by two or three community sessions that were modelled on the training workshops and were run at school for parents.

The community sessions were, ideally, organised by the lead team with support from the facilitator. The dates and times for the community sessions were decided by the lead team and the school. The incorporation of languages from the community could be achieved by running the sessions in the appropriate first language or by parents working in language groups as required. The suggested content of the sessions derived from the Numeracy Development Projects (NDP) and involved counting strategies, part–whole thinking, grouping, place value, multiple strategies, sharing ideas, visualising, and developing number sense.

Methodology

Research Questions

The aim of this study was to explore the impact of the community sessions on parents and their children and on teachers and the school. This involved canvassing the views of parents about their involvement in the HSPN pilot and the impact on their children's attitude towards and learning of mathematics. It also incorporated the perceptions of facilitators and lead teachers about the home–school partnership model.

The specific research questions that were addressed were:

1. How well do the processes of the HSPN deliver intended outcomes to parents and families?
2. In what ways is the programme perceived to have an impact on the students' attitudes to and learning of mathematics in and out of school?
3. In what ways is the programme impacting on the partnership between the school and parents and families?
4. What are key areas of the HSPN programme that require further exploration or evaluation?

Case Studies

A case-study approach was used to gather in-depth information from the different groups of people involved in the programme. We visited three schools (called A, B, and C) for the case studies. In addition, we interviewed two lead teachers from a fourth school (D) and attended their training workshop. This school did not continue with the community sessions, so further interviews were not conducted. Auckland and Wellington were chosen as the regions from which to select schools because of their large proportion of Pasifika students.

Community sessions

It was planned to visit two community sessions at each of the three case-study schools[1]. The purpose of the first visit was to build relationships with the parents, school staff, and facilitators and to introduce the scope and the purpose of the study. Observations of the content of the session and the interactions between the parents, teachers, and facilitators were also made. The second visit was similar, but interviews were conducted with volunteer parents, lead parents, and lead teachers.

In all, 11 volunteer parents, two lead parents, and five lead teachers were interviewed. The parents interviewed were identified as Sāmoan, Tongan, Cook Island, Niuean, Māori, Dutch, Ethiopian, and New Zealand European. All lead parents and teachers were female, as were all but two of the 13 parents, although men made up about one-third of the parents at the sessions. The selection of people was based on the participants' availability and willingness to participate.

Survey

Facilitators were mailed a survey about their views of the workshops, community sessions, roles, impact of the NDP approach to mathematics, and issues for implementing and sustaining the programme. Two-thirds of the surveys were returned and analysed.

Results and discussion

Implementing the Programme: the Training Workshops

The purpose of the workshops was to prepare the lead team for running a series of six community sessions for parents. The workshops were often clustered with other schools to provide support and opportunity for sharing with other lead teams. The workshops, to plan the next three community sessions, involved the lead teachers for the whole day and the lead parents for the afternoon.

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[1] In school B, a significant community event fell on the day planned for the final session, so only the interviews were conducted.

The feedback from lead parents, most of the lead teachers, and the facilitators all suggested that the training workshops prepared them well or extremely well for the community sessions. Lead teachers from all schools found the resources and activities useful and relevant. Lead teachers and lead parents expressed how good it was to hear the experiences of lead parents from other schools. Lead teachers also benefited from sharing with other lead teachers and commented that it was useful to hear what other schools were doing.

Lead teachers and facilitators identified some potential ways to improve the workshops so that they would cater better for parents. These are outlined below.

- *Pace:* A number of facilitators and some lead teachers commented that the pace of the workshops was too fast. Several facilitators stated that more time could be spent on reflection. As one facilitator suggested, "Do less, well."
- *Content:* One concern identified by facilitators and some lead teachers was the difficulty of the content in the workshops. Although lead teachers from all four schools commented that "the games were engaging", the activities were useful, and "the take-home pack was good", some facilitators and lead teachers identified that the content difficulty of the workshops could be better managed to cater for the range of parents' mathematical dispositions.
- *Number Framework stages:* Some facilitators and lead teachers also commented that there was a need to make the Framework stages more understandable for lead parents, so that they could pass this knowledge on to other parents. One facilitator stated:

  > Make the delivery and content as clear and as simple as possible ... Once this is established, extension and embellishment seem to follow.

- *Partnership and roles:* Facilitators had a variety of ideas about balancing the contribution from lead parents and lead teachers at the workshops. One suggestion was to bring the lead parent and the lead teacher to the workshops at the same time so that they feel they are an equal part of the lead team, thus supporting the relationship building.

Implementing the Programme: Community Sessions

The purpose of the community sessions was to introduce parents to the Number Framework, NDP language, and classroom delivery. This was done by exploring and discussing the activities and then relating them to the developmental stages of the Framework. The sessions involved a combination of whole-group introduction, activity-based learning, sharing, and small-group work. Almost all of the facilitators agreed or strongly agreed that the sessions were effective for parents, families, teachers, and lead parents. Similarly, all parents and lead parents were very positive about the sessions. Most parents identified the welcoming atmosphere, the mathematics activities, and the fun of the sessions as three features that worked well.

Timing of the community sessions

The timing of the community sessions appeared to be an important factor in determining the number of parents available to attend and was influenced by local factors, the time of the year, and local events. All schools were running these sessions in late winter, so 6:00 p.m. was not a popular time for parents. Schools A and C found 1:30 p.m. a better time for getting parents. These afternoon sessions would finish at 3:00 p.m., and the parents could then take their children home. School A also held successful meetings at 9:00 a.m. The best time to include as many parents as possible seemed to depend upon many factors. These are best discovered by schools sending a brief survey to parents asking what times would be convenient.

Finding lead parents

Essential to the HSPN is the lead team: a lead teacher and a (bilingual) lead parent. This is described as "crucial to the successful implementation of the programme" (Ministry of Education, 2003, p. 16), providing the "essential links" between the home and school. Most schools asked around their staff and then approached potential lead-parent candidates based on recommendations. Approaching a known parent seemed to achieve a more successful result than an advertisement in the school newsletter. School D had difficulty finding sufficient lead parents. The lead teacher approached a number of parents with previous HSPL experience, but they were reluctant to participate because the focus was mathematics. This lack of lead-parent leadership resulted in school D discontinuing the community sessions.

Parents attending the community sessions

Getting parents to attend sessions was the first of a number of challenges because without parents, there is no one to have a partnership with. One facilitator stated "attracting new and different parents" was an important issue for sustaining the HSPN. The HSP resource folder provides a range of ideas for promoting the programme, such as advertising through the local radio, placing an advertisement in the local paper, involving local leaders, and setting up a phone tree. The most common ways that the community sessions were communicated to the parents was by school newsletter, notices, and children writing letters home. Other ways included a phone call from the lead parent or being informed about it at a previous parent meeting. One key way to get parents attending was through the lead parent. At school A, the lead parent approached parents at their church, telephoned, and talked to them on the street to let them know about the sessions. This type of lead-parent "network" was also promoted by the HSPL. Other classroom teachers in two schools used their students to network by reminding their classes of community sessions.

Incentives

Schools used a variety of incentives to encourage parents to attend their sessions. All four schools had prize draws after each session. At one school, a brightly coloured notice was sent home with the children. Parents could use the notice as a ticket in a prize draw at the community session. Many parents stated that the prize draw and the food were things they liked about the sessions. Including children was another successful way to get parents attending. Children were present at the sessions for a range of purposes: they performed a welcome, modelled mathematics done in class (set up as learning stations), or participated in the sessions with their parents (as recommended for two out of the six sessions). School A began each community session with a performance from the children. The parents we observed noticeably enjoyed this. The primary reason parents were at these sessions was for their children, and so it makes sense that involving their children was a positive influence in getting parents to attend.

Sharing

Most parents identified sharing their experiences and ideas with other parents as a successful feature of the community sessions. They found it helpful to hear what other parents had tried and about similar problems they might be having. This sentiment was echoed by lead parents about the workshops.

The Roles of Lead Parent, Lead Teacher and Facilitator

The facilitator

Our findings revealed that building relationships with parents was an important aspect of the facilitator's role that could be highlighted more in the HSP resource book. Their role is described as preparing the school to undertake the programme, training the lead team (in the training workshops), and providing guidance and support for the lead team (in the community sessions). Additionally, lead teachers and a number of facilitators noted that it was also important for the facilitator to build relationships with the lead parents and other parents. One lead teacher identified that her facilitator's concern for "relationship building before the maths" was an important aspect of their role. Another noted that their facilitator modelled the two-way relationship needed for the community sessions by listening and responding to lead parents' ideas. This was in addition to modelling, "dealing with tricky questions", and talking about the importance of parents for their children's learning.

The lead team

The choice of the members of the lead team is important. Almost all facilitators reiterated the critical role of the lead team or the lead teacher. One facilitator said:

> Lead teachers are pivotal to the success of community sessions ... Training workshops carefully planned to meet their needs are essential.

Three of the lead teachers described their responsibilities in the community session as providing the structure and organisation and deciding what happens in the sessions. The lead teachers described the lead parent's role as liaison with parents, "facilitating, going around the groups", and "introducing the activities". One lead teacher identified that it was important for lead parents to "make connections with other parents" and that this was needed for them to return. A lead parent described their role as introducing the activities, explaining to parents, and joining in with the other parents.

Changing roles as time went by

Our observations of the community sessions indicated that developing the lead parents as leaders changed over time and was dependent upon the skills and competencies of each person and what they felt confident to do. Some facilitators recommended adapting the community sessions by giving "more responsibility" and opportunities for "more input from lead parents".

In schools A and B, we noticed a change in the dynamic of how the sessions were being run. At the earlier sessions, the facilitators took a more up-front role in leading and presenting. In the latter sessions, the facilitators had handed over the leadership role to the lead team and stepped back into a support role.

In school A, this handover of leadership was predominantly to the lead parents. From the beginning of the earlier community session, the lead parents had stood up briefly to introduce and explain several activities. During our second visit, we observed that the lead parents were taking an even more active role and they appeared more confident presenting activities. We were also informed by the lead teacher that the lead parents had been involved in selecting and planning the activities. The lead teacher identified that encouraging the "lead parent [to] take control of the direction of the session" was working well and was important for the success of the programme.

At school B, the leadership of the sessions was handed over to the lead teacher and the lead parents were almost indistinguishable from other parents. They were, however, pre-informed about what was happening and positioned at table groups to support discussion. School C had a similar dynamic, where the lead teacher led the session, with the small-group facilitation conducted by the lead parents, but there was no visible evidence of "handing over" the leadership or decision making to the lead parents.

Building lead parent confidence

The HSP resource folder material states that the lead parent has, ideally, amongst other criteria, the confidence or potential confidence to co-lead and introduce and model activities. From the interviews with lead parents and lead teachers and surveys from facilitators, it was very evident that many parents who were selected for the role of lead parent gained in confidence. One facilitator stated, "[I] love how parents have grown in confidence and maths understanding." A lead parent from school A stated that she "used to be shy, but [is] confident now". Some lead parents also overcame the fear of coming into school, of talking to large groups, and of mathematics itself. As well as gaining in confidence, many parents developed their knowledge of numeracy. One lead parent had a complete turnaround, from being not interested in school to passionately getting involved, and she attributed her gain in confidence to participation in the programme:

> If I didn't get involved ... I wouldn't have the courage to stand up and talk in front of people and to open up and help others.

She was also very keen to step into more of a leading role than greeting, sharing, and working with the small groups.

Lead teachers and facilitators noted that a number of lead parents went on to become teacher aides, gain employment, or even begin teacher training as a result of their involvement with the HSPN. This empowerment of the lead parent was a notable outcome of the HSPN and may also play an important part in sustaining the connection between the community and the school.

Need for explicit information about the lead-parent roles

The HSP resource folder material indicates the importance of stating clearly what the lead parent's role is expected to be: co-leadership. It identifies the role of the lead team as planning and organising the community sessions and maintaining contact with the community. However, there is no explicit statement describing the separate roles of lead teacher and lead parent. In one school, some of their lead parents were not aware of their role for the community sessions and subsequently lacked the confidence to grow into co-leadership. This highlights the need for more discussion about the role of lead parent as co-leaders. The lead parent and lead teacher are vital to the HSPN, so it would seem important to not only make the specific expectations of their role more explicit at the time they are recruited but also to identify the benefits of participating in the HSPN and the opportunity for personal growth.

Supporting First Language

Two features of the HSPN are the facility to incorporate parents' first language into the sessions and the flexibility for schools to adapt the programme to cater for their community. Most parents involved in the programme had a first language other than English, and consideration of this when promoting and running the community sessions may avoid overwhelming parents with too much information. A balance of oral and written key messages seemed to be effective for introducing and supporting parents. How first languages were catered for was worked out by the lead team, with guidance from the facilitator, principal, and community. The model suggested for the HSPN programme is to have each language group supported by a bilingual lead parent.

All three case-study schools ran their sessions with the introduction and modelling done predominantly in English. In all community sessions we observed, parents chose where they sat and which groups they worked in. In one school, the parents were asked if they would be interested in grouping into language groups. They said they would rather mix with other people, but during the sessions, most of the Tongan parents gradually gravitated together to share and discuss things in Tongan. In another school, some of the Sāmoan parents grouped themselves together and spoke mainly in Sāmoan when they were working on the different activities. The lead parent at the same school considered it was important to give parents the opportunity to speak in their first language and further argued that English is one barrier and mathematics is another. Together, they could be an intimidating reason not to come to the sessions. She noted, "Some [parents] can't make it [because] they are too embarrassed about the language." Parents from school B also agreed and commented that some parents were "scared of English". This illustrates the importance of having a bilingual lead parent for each language group (the HSPN model) and providing support for the different languages in the school.

Whole-school Involvement

Engaging in whole-school professional development is associated with positive learning outcomes for students (Timperley, Parr, & Higginson, 2001). The HSP resource folder material states that "all staff need to take ownership" of the programme (Ministry of Education, 2003, p. 17) and recommends that "all members of the school community are kept informed and are involved with the programme" (p. 8). It also suggests including the principal, other staff in the school, and other parents who may not be attending for whatever reasons. It is hoped that this wider involvement of the school and the community will provide:

- opportunities to share the workload of the lead team with other staff
- better school-wide promotion of the programme because the classroom teacher is more informed
- more visible people to develop and support the partnership
- raised awareness of the programme within the school and in the community.

However, at the schools we visited, the school staff had various degrees of ownership of the programme and attendance at the community sessions.

The HSP resource folder material states that implementing the home–school partnership involves the parent and the classroom teacher; further, that teachers learn by:

> getting a better understanding and insight into the backgrounds, cultures, and home numeracy practices of the children they teach.
>
> (Ministry of Education, 2004, p. 7)

This indicates the importance of the teacher learning about the child and the parent and appropriately incorporating what they have learnt into the classroom programme. For that to happen, both the parent and the classroom teacher need to be involved in the HSPN in some way. It follows that the HSPN should be a whole-school initiative, otherwise the relationship building may be limited to only those who attend the community sessions.

The HSP resource folder material reinforces the need for the principal's attendance or endorsement. Their absence may suggest that the programme is a low priority and therefore the relationship with parents is also a low priority (Gorinski, 2005). In three of the schools, the principal was involved in the programme, attending or making an appearance at the sessions. School D was initially signed up for the programme by a principal who had since left the school. The responsibility for implementation fell to two teachers who were not previously aware of the programme. This may have contributed to the programme being less successful at this school.

Parents and the Number Framework

The community sessions were about experiencing and exploring mathematics and learning through doing, discussing, and enjoying. The mathematics used was derived from everyday mathematics used in the home or the community and was about making connections to prior learning and real situations. One of the key messages was that "maths is everywhere"; the HSP resource folder material includes an extensive list of examples of "daily life maths" (Ministry of Education, 2004, p. 51). The range of learning strategies, the use of materials and visualising, the sharing, and discussion involved in the community sessions address a significant number of the attributes identified for quality teaching of diverse learners (Alton-Lee, 2003). Overall, parents responded positively to the mathematics in the sessions. They noted that the mathematics was fun, more interesting and engaging, and helped develop their understanding of the learning of mathematics. One parent stated the mathematics was "more understandable".

Traditional mathematics and the NDP approach to mathematics

The community sessions focused on contemporary classroom mathematics and therefore were not likely to endorse parents' own experiences and ideas about mathematics. Research suggests that this difference between the mathematics parents know and the mathematics their children bring home could lead to frustration (Eyres & Young-Loveridge, 2005) and that when home and school practices are significantly different, there could be negative effects on children's achievement (Wylie, Thompson, & Lythe, 1999). It was recognised by almost all parents and teachers that the mathematics content in the sessions was very different from parents' expectations. Most parents interviewed described their own mathematics experience as what might be called traditional mathematics: the vertical algorithm using renaming or carrying and borrowing. A few parents had initial concerns about the NDP approach to mathematics. They commented that they were trying to teach their old mathematics when the children were bringing home "new maths". One parent from school B described her reaction to the mathematics at the sessions:

> At first I thought this was a waste of time because they were teaching maths in a way that I was not taught when I was at school. This made it hard to agree with how things were added or multiplied ... It was trying to change old habits that made things frustrating.

Despite the potential for parents to feel frustrated about their concept of mathematics being challenged in the sessions, almost all the parents in our study got involved with the mathematics activities and started to develop their understanding. One parent stated,

> [I was] scared to come at first ... shamed because [I'm] not good at maths, but now [I'm] very happy.

Many parents stated that as a result of the sessions, they now know what they can do to help their children with numeracy. One parent stated it was "clear understandable maths".

The language of mathematics

The sharing in the community sessions exposed parents to mathematical discussion and supported them to begin learning the language used in the NDP. This is important because the NDP employ some different mathematical language from traditional mathematics, with strategies such as part–whole, halving and doubling, place value partitioning, and tidy numbers. This alignment of language and understanding enabled parents to ask more targeted questions about the strategies or knowledge children had, thus getting parents and teachers to focus on the same thing, children's learning. As a result, one lead teacher stated that parents were more confident to say what they want to see and were looking at a "deeper level" of mathematics.

Influencing Children's Learning

Parental influence

One of the goals for the HSPN is that parents recognise that they are an important part of their children's success at school. In the HSP resource folder, "You are your child's first and most important teacher" (Ministry of Education, 2004, p. 40) is one of the key messages. There is significant literature that argues the importance and benefits of parental involvement (Alton-Lee, 2003; Biddulph et al., 2003; Eyres & Young-Loveridge, 2005; Merttens, 1999). Parents can influence how children utilise their time and are therefore important influences on their children's learning at many levels, directly and indirectly, not only by spending quality time with their children but also by their influence on a significant amount of children's time outside of school for activities such as holidays, television viewing, and many other experiences (Biddulph et al., 2003).

Almost all lead parents, lead teachers, and parents stated that parents were important in influencing their children's learning. Parents were clearly aware of this message, with one commenting that "parents are the first teachers."

Influence of the HSPN

Although no children were interviewed, parents and teachers were asked about their perceptions of the impact of the HSPN on their children. Many parents from all three schools made a range of comments such as the children were "happy to do maths now", the games improved children's knowledge, the activities made mathematics easier, children were working more easily, and they were having more discussion about mathematics at home. Others were not sure about the impact of this approach to mathematics and commented that there were "still problems with their times tables" and that it was "harder to know if they're doing better". The latter point should be expected, as the NDP approach to mathematics is not about final marks but about strategies and understanding and is, accordingly, harder to quantify.

Two lead teachers noted differences in students that they attributed to the HSPN. They commented that students had better attitudes towards mathematics, improved self-esteem, less fear of making a mistake, and a "definite improvement on attainment". One lead teacher had used a diagnostic test before and after the community sessions, had noted improvements, and attributed these to the HSPN. At three schools, we heard a number of anecdotal statements about children who previously were not interested in learning and who had changed their attitude and were participating with more enthusiasm because their parents were involved. Parents from school B stated that their children were using the mathematics strategies more effectively and that they were solving mathematics problems faster as a result.

Developing the Partnership

The HSP resource folder material defines a mutually beneficial partnership as one in which:

> teachers learn about the children's language and culture and how to incorporate this prior learning in school programmes. The parents learn the culture of the school, its processes and its expectations.
>
> (Ministry of Education, 2003, p. 8)

This indicates the importance of both the parent and the teacher learning from each other.

At the community sessions, there was opportunity for contribution, sharing, and feedback. Parents, teachers, and facilitators described the environment as relaxed and comfortable, one in which parents began to feel they could share their answers with other parents in their small group, as well as with the lead team and the facilitator.

These community sessions seemed to offer what the parents wanted and were comfortable with. They found out about mathematics going on in the classroom and how they could work with their children. However, these aspects of the community sessions, although open and helpful to parents, did not appear to encompass the attributes of a two-way partnership. Lead teachers from one school described one of their sessions as being more like a "parent-information evening" than a partnership evening. In another school, the idea that teachers teach and parents practise at home what the children have learned in school was reinforced in the community sessions. Two parents from school A stated that, although the programme was called home–school partnership, the relationship was not a partnership. They acknowledged that it was a positive step in that direction, but that "we're not there yet." The HSPN goal of "sharing and working together" could be seen as an intermediary step towards the goal of "establishing a partnership".

Essentially, none of the parents or lead parents interviewed had come across the NDP approach to mathematics and, accordingly, the topics were new to them. This "intermediary step" of informing parents gives them some understanding about contemporary teaching of mathematics; from this, they can begin dialogue with the classroom teacher. Many parents stated they were starting to feel more confident with mathematics and were developing the understanding and vocabulary to ask questions about learning.

A genuine, two-way partnership involves developing a shared vision, mutual respect, making decisions together, and sharing the responsibility. These attributes were not fully apparent at any of the three case-study schools. Evidence from our observations of the community sessions, interviews and conversations would indicate that there were beginnings of a partnership to varying degrees in all four schools. Research indicates that developing this into a genuine partnership is likely to take time (Merttens, 1999).

Issues for Sustaining the Programme

Encouraging Parents into the Community Sessions

All four schools (A, B, C, and D) developed a range of ways to encourage parents to attend the community sessions. However, although some of the initial community sessions had 60–100 parents attending, subsequent sessions had well under half those numbers. This variation in parental attendance may indicate that there is still room to develop further strategies to continue building parent involvement and highlights the need to ensure that the sessions are still useful for the parents who attend sporadically. In three of the case-study schools, the number of parents who did attend was a small proportion of the parents that could attend in each school. There could be a range of reasons for this: content, timing, fear of mathematics, fear of English, local events, or general busyness. If the HSPN is intended to be an ongoing programme, getting parents to attend the sessions is vital. Schools may need to identify possible barriers that parents may have to coming into school. If the community sessions meet the needs of parents and make them feel welcome, respected, and valued, then it makes sense that they will come again or at least share their experiences and encourage other parents to attend subsequent sessions. The school could also include the lead parents in the campaign to promote the HSPN and utilise their connections to the community. Essentially, schools need to continue developing ways to reach their community.

Retaining the Skills, Experience, and Relationships

If the first partially-funded year of the HSPN is regarded as "training for the lead team", it follows that this team should be involved in the subsequent year. Many facilitators identified the importance of retaining the experience of the lead parent and lead teacher. One facilitator stated, "The biggest problem appears to be the selection and retention of lead parents." Most of the lead teachers commented that supporting the lead parents was important for sustaining the HSPN. It follows that the lead parent is a fundamental part of the HSPN because they provide the link between the school and the community. This parent "liaison" was identified as important for maintaining momentum and sustainability (Gorinski, 2005).

If the HSPN is to be an ongoing programme, then it makes sense to utilise and build on the experience gained for the subsequent years. Some facilitators also identified the importance of the continued involvement of the facilitator to support the programme. This may entail the facilitators maintaining the relationships they had built on an ongoing basis.

Funding the Programme in the School

Several facilitators made the point that the workload exceeded the time allocated, and others indicated that time for planning and preparation was an important issue. Lead teachers acknowledged that their role involved a lot of work. Although having more teachers involved on the programme may help to share the workload, getting classroom teachers to attend the school-time community sessions would require significant funding and resources. Of the four schools visited, only one school "required" staff to attend the sessions. Notably, these were all evening sessions and did not involve organising and funding release time for teachers. These aspects highlight the need to balance the involvement of school staff with the development of the partnership with parents, yet still remain affordable and sustainable for the school.

Ongoing funding of the HSPN is the responsibility of the school. The HSPN pilot was partially funded by the Ministry of Education on the understanding that it was for the start-up of the programme, that the school would also contribute, and that subsequent years of the programme would be supported by the school and the community. Although all schools involved in the HSPN are made aware of this, the reality of providing funding from the school budget may be problematic. A number of facilitators stated the funding for release time for lead teachers and funding for lead parents was important for sustaining the HSPN. Many facilitators identified the importance of allowing sufficient time for the lead team to plan. It seems likely that schools may still welcome further support to maintain the HSPN until it becomes an integrated part of their school culture, which may take years. This support could involve some continuing facilitator support, funding for the lead parents, or one-off grants for resources.

Further Exploration

This study was a small exploration into the HSPN in three schools. The schools were not necessarily representative of all the schools involved in the HSPN. Each school was in its first year of the HSPN, so sustainability issues can only be predicted rather than experienced. Therefore, there are limits to generalising the findings.

This initial study suggests that the following investigations could be conducted to extend the study further:

1. Explore how successfully the HSPN continues to operate in subsequent years.

2. Involve a wider range of schools to make the study more representative. Select several new case-study schools or survey all schools in their first year of the HSPN.
3. Include feedback from stakeholders not formally included in the 2006 evaluation, such as all the parents and lead parents who attended the community sessions, the parents who did not attend the sessions, and children of parents who attended community sessions.
4. Because the programme is premised on partnerships and parent involvement leading to improvement for student learning and achievement, explore the development of more direct measures of student outcomes over a longer term.

Conclusion

Underlying the HSPN were five outcomes. These outcomes were used as points of reference for this study. A brief summary of the evidence we collected in relation to each outcome follows.

Reinforce the fact that parents and families are one of the greatest influences on children's learning and development and are essential to their success at school.

The key messages throughout the community sessions reinforced the influence that parents have over their children's learning. All parents understood this message and stated that they were an important influence on their children's learning.

Endorse what families and teachers are already doing for children's numeracy development.

The HSPN endorses parents spending time with their children and using the NDP approach to mathematics. These sessions did not endorse parents' prior experiences and understanding of school mathematics. However, our findings suggested that although there was initial hesitancy about the NDP approach, it was not the issue it could have been. This is likely to be because the mathematics in the programme is "real life" and parents found it accessible, inclusive, engaging, and helpful. They took this mathematics home and began to integrate it into their home practice.

Increase parents' and families' understanding of the NDP approach to mathematics and practical ways of helping children learn.

At the sessions, parents were provided with a wide range of practical activities and encouraged to get involved with their children and to use and adapt what they had learnt. Parents noted their own increased confidence to do mathematics with their children. All case-study schools gave out packs of activities and support materials for parents to take home. These were very popular, and parents stated they were using them at home.

Share ways in which families and teachers working together can make an even greater impact on children's numeracy development.

The NDP mathematics that parents learnt in the community sessions aligns with the mathematics that teachers are likely to be using in the classroom. A number of parents described using these activities at home and noted an improvement in their children's achievement and attitude. They also stated they had more confidence to talk to the teacher about their children's learning.

Establish a caring working partnership between school staff and the community.

The three case-study schools (A, B, and C) had progressed in different ways, but all exhibited some fundamental elements of a partnership: respect, inclusion, and developing a shared vision. To achieve the partnership advocated by the HSPN, teachers need to actively seek knowledge about their parents' and children's home numeracy practices and to incorporate it in their teaching and learning programme in the classroom. This aspect of a partnership was not obvious in any of our interviews, surveys, or conversations in the case-study schools. However, our findings suggest that it is likely that the HSPN is a first step that could support this partnership to occur.

Success Factors and Enhancements

The HSPN model is founded on a number of aspects of good practice that are likely to have a positive impact on learning: catering for diverse learners, family involvement in education, and learning for understanding in mathematics. The evidence we collected from this small-scale study gives some indication that the goals for the HSPN were either achieved or were being worked towards. Key factors that were identified as important for successful implementation of the HSPN involving content, processes, and people were:

- careful consultation and selection of the lead parent
- support from the school community and school leadership
- sharing the leadership with the lead parent and supporting the lead team into the role
- developing marketing strategies appropriate to the school and the community
- ensuring the community sessions are social and enjoyable and engaging to parents
- mathematical exploration that is accessible and relates to life
- providing a flexible HSPN structure that the school and community can adapt to suit their needs.

Some of the enhancements that were identified were:

- incorporating and respecting parents' contributions to develop a genuine, two-way partnership
- developing more opportunities to support community sessions in first language
- having a succession plan to ensure continuity, including retaining the experience of the lead team
- fully informing the lead team, the school, and the community about the scope and purpose of the HSPN
- developing further ways to maintain attendance of the HSPN, for example, reaching parents who did not come or researching when parents can attend.

In general, most of the lead teachers from the case-study schools and most of the facilitators considered that the HSPN was a successful model. Similarly, all the parents and lead parents we talked to or interviewed rated the programme as a success. Any earlier fear of mathematics or tensions between traditional mathematics and the NDP approach to mathematics were quickly resolved because almost all parents realised the accessibility and benefits of exploring and sharing mathematical ideas. Almost all parents finished the sessions with increased confidence in doing mathematics with their children. For lead parents, the sessions could have additional benefits. Facilitators and lead teachers described a number of situations in which lead parents were empowered through the HSPN.

Each case study school was unique and had individual challenges for implementation of the HSPN in 2006. They each had a different community, different people, different relationships, and a different mix of cultures to accommodate when developing a relevant and useful series of community sessions. The HSPN model has the flexibility for schools to adapt the programme to their needs and the needs of their community.

The HSPN is about people and getting people together to grow relationships into partnerships. "With your food basket and my food basket, everyone will have enough."

Nāu te rourou, nāku te rourou, ka ora te iwi.

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Appendices

Appendix A (Patterns of Performance and Progress)

Composition of the Year 5–9 Cohort 2002–2006

Year	2002	2003	2004	2005	2006	2002	2003	2004	2005	2006
	Year 5					**Year 6**				
Number of students	*11316*	*18665*	*9868*	*8353*	*5830*	*11818*	*19338*	*9959*	*8689*	*6206*
Ethnicity										
European	58.0	59.7	59.1	62.2	63.4	57.6	58.9	59.2	63.0	63.0
Māori	22.9	23.0	19.5	19.0	18.5	23.4	23.5	18.2	18.2	17.2
Pasifika	9.7	8.6	10.9	8.7	7.3	9.4	8.5	11.2	7.9	7.1
Asian	4.9	4.5	5.9	6.0	5.8	5.2	4.8	6.4	6.2	7.1
School Decile										
Low decile	34.9	34.1	29.4	21.9	19.3	34.2	33.8	29.7	21.2	17.5
Medium decile	38.0	37.5	38.9	38.8	35.6	39.3	38.5	37.4	38.7	35.6
High decile	27.0	28.4	31.8	39.3	45.1	26.5	27.7	32.8	40.1	46.8
	Year 7					**Year 8**				
	6418	*13460*	*8374*	*6348*	*9515*	*5802*	*11796*	*7306*	*5911*	*8853*
Ethnicity										
European	56.1	53.3	58.9	64.7	65.7	57.9	54.0	58.2	63.9	66.1
Māori	26.3	28.1	24.5	21.5	18.7	25.7	27.4	24.6	21.2	19.0
Pasifika	8.8	10.4	9.5	6.2	6.9	8.5	10.8	10.0	7.4	6.3
Asian	4.9	4.5	3.6	3.8	4.3	4.9	4.3	3.4	3.7	4.4
School Decile										
Low decile	32.5	41.3	28.4	12.5	17.1	33.0	42.4	31.5	15.7	17.2
Medium decile	47.1	43.4	49.3	55.5	41.9	47.6	41.6	48.5	55.0	43.4
High decile	20.4	15.3	22.2	32.0	41.1	19.4	15.9	20.0	29.3	39.3
	Year 9									
				4068	*6740*					
Ethnicity										
European				66.0	62.3					
Māori				20.3	19.4					
Pasifika				4.8	9.9					
Asian				3.8	3.7					
School Decile										
Low Decile				11.3	14.0					
Medium Decile				51.5	60.8					
High Decile				37.2	25.2					

Appendix B (Patterns of Performance and Progress)

Percentages of Year 2–9 Students at Framework Stages on Each Domain (Initial and Final) in 2006

	Year	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
	Number of students	*5177*	*5161*	*5744*	*5830*	*6206*	*9515*	*8853*	*6740*
	Additive Domain								
	Initial								
0	Emergent	3.1	0.6	0.2	0.1	0.1	0.1	0.1	0.1
1	1:1 Counting	12.3	4.4	1.1	0.5	0.3	0.1	0.1	0.1
2	Count from One w. Materials	43.5	18.9	5.8	2.2	1.1	0.7	0.3	0.3
3	Count from One w. Imaging	21.3	17.6	6.6	2.7	1.6	1.0	0.7	1.0
4	Advanced Counting	17.2	44.6	44.8	33.7	25.1	20.3	15.7	14.1
5	Early Additive P–W	2.4	13.3	36.6	46.8	47.6	46.2	40.0	44.7
6	Advanced Additive P–W	0.0	0.6	4.8	13.3	22.0	26.5	33.4	32.0
7	Adv. Multiplicative P–W	0.0	0.1	0.1	0.7	2.2	5.0	9.7	7.7
	Total stages 6–7	***0.0***	***0.7***	***4.9***	***14.0***	***24.2***	***31.5***	***43.1***	***39.7***
	Final								
0	Emergent	0.4	0.1	0.0	0.1	0.0	0.0	0.0	0.1
1	1:1 Counting	2.3	0.7	0.2	0.1	0.1	0.1	0.0	0.1
2	Count from One w. Materials	16.4	5.7	1.6	0.8	0.3	0.3	0.2	0.1
3	Count from One w. Imaging	21.4	8.8	2.2	1.1	0.6	0.5	0.2	0.3
4	Advanced Counting	44.7	44.0	29.7	18.2	10.7	8.8	6.4	5.4
5	Early Additive P–W	14.3	36.6	50.1	46.0	39.1	35.5	27.8	29.5
6	Advanced Additive P–W	0.4	3.9	15.3	30.2	40.0	39.2	39.9	43.5
7	Adv. Multiplicative P–W	0.1	0.2	0.9	3.6	9.2	15.6	25.4	21.0
	Total stages 6–7	***0.5***	***4.1***	***16.2***	***33.8***	***49.2***	***54.8***	***65.3***	***64.5***
	Multiplicative Domain								
	Initial								
n/a	Not entered or applicable	88.3	52.0	19.0	8.7	5.6	3.7	2.0	0.4
2–3	Count from One	6.6	17.9	15.4	10.3	6.8	4.4	2.7	1.7
4	Advanced Counting	4.6	25.1	42.7	36.2	27.2	21.1	14.9	14.1
5	Advanced Counting	0.4	4.3	17.1	27.6	30.8	30.0	26.8	28.2
6	Early Additive P–W	0.1	0.6	5.2	14.2	21.6	28.0	32.4	32.0
7	Advanced Additive P–W		0.1	0.6	2.9	7.4	11.1	17.5	17.8
8	Adv. Proportional P–W				0.1	0.6	1.7	3.7	5.8
	Total stages 7–8	***0.0***	***0.1***	***0.6***	***3.0***	***8.0***	***12.8***	***21.2***	***23.6***
	Final								
n/a	Not entered or applicable	56.3	21.5	7.9	4.0	3.5	1.3	0.6	0.2
2–3	Count from One	11.6	10.6	6.2	3.0	1.5	1.4	0.8	0.6
4	Advanced Counting	26.5	43.4	35.5	22.2	12.5	10.4	6.3	5.4
5	Advanced Counting	4.9	19.2	29.9	31.8	25.6	22.6	16.9	16.5
6	Early Additive P–W	0.6	4.6	16.9	27.6	34.0	34.2	34.4	34.6
7	Advanced Additive P–W	0.1	0.7	3.6	10.7	19.9	23.4	29.1	29.0
8	Adv. Proportional P–W		0.0	0.1	0.8	3.0	6.7	12.0	13.8
	Total stages 7–8	***0.1***	***0.7***	***3.7***	***11.5***	***22.9***	***30.1***	***41.1***	***42.8***

Appendix B – continued

Percentages of Year 2–9 Students at Framework Stages on Each Domain (Initial and Final) in 2006

Year		**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Number of students		*5177*	*5161*	*5744*	*5830*	*6206*	*9515*	*8853*	*6740*
	Proportional Domain								
	Initial								
n/a	Not entered or applicable	88.1	53.1	20.5	12.4	6.9	5.8	3.5	0.7
1	Unequal sharing	4.3	12.3	11.6	8.3	6.2	3.7	2.5	0.8
2–4	Equal sharing	7.4	31.1	49.0	43.1	35.9	28.7	21.2	17.8
5	Early Additive P–W	0.2	3.1	15.4	25.1	28.4	28.0	26.0	30.9
6	Advanced Additive P–W	0.0	0.3	2.9	8.7	15.0	20.9	24.2	16.4
7	Adv. Multiplicative P–W			0.5	2.4	7.0	11.4	19.1	29.1
8	Adv. Proportional P–W		0.0	0.0	0.0	0.5	1.6	3.6	4.3
	Total stages 7–8	***0.0***	***0.0***	***0.5***	***2.4***	***7.5***	***13.0***	***22.7***	***33.4***
	Final								
n/a	Not entered or applicable	56.5	21.7	8.5	4.6	4.4	2.1	1.7	0.8
1	Unequal sharing	6.5	6.9	2.5	1.7	0.9	0.5	0.5	0.2
2–4	Equal sharing	33.5	51.5	43.8	28.9	17.4	14.7	9.7	6.9
5	Early Additive P–W	3.1	15.9	30.4	33.3	29.2	26.5	19.7	24.0
6	Advanced Additive P–W	0.3	3.3	11.7	21.0	27.3	25.9	26.5	18.6
7	Adv. Multiplicative P–W	0.0	0.6	3.1	10.0	18.2	24.2	30.9	37.7
8	Adv. Proportional P–W		0.0	0.1	0.5	2.7	6.1	11.1	11.8
	Total stages 7–8	***0.0***	***0.6***	***3.2***	***10.5***	***20.9***	***30.3***	***42.0***	***49.5***
	Fractions								
	Initial								
n/a	Not entered or applicable	89.6	53.4	22.0	11.4	6.0	4.9	3.6	3.0
2–3	Unit fractions not recognised	9.1	33.6	39.1	27.0	17.7	9.3	6.3	4.3
4	Unit fractions recognised	0.9	9.4	21.8	25.1	23.9	20.8	18.0	12.5
5	Ordered unit fractions	0.3	3.5	16.0	30.7	38.4	41.4	37.7	39.9
6	Co-ord. num’rs & denom’rs		0.0	1.0	4.7	9.4	14.2	17.4	20.7
7	Equivalent fractions			0.1	1.0	3.1	6.6	10.9	15.6
8	Ordered fractions			0.0	0.3	1.4	2.9	6.1	4.1
	Total stages 7–8	***0.0***	***0.0***	***0.1***	***1.3***	***4.5***	***9.5***	***17.0***	***19.7***
	Final								
n/a	Not entered or applicable	60.0	24.5	12.5	8.4	6.8	2.9	1.9	11.1
2–3	Unit fractions not recognised	20.4	21.2	13.2	6.2	3.6	2.3	1.6	0.8
4	Unit fractions recognised	13.1	28.2	26.0	18.8	13.7	11.3	8.0	5.7
5	Ordered unit fractions	6.4	24.4	40.1	45.0	40.7	36.5	29.3	26.1
6	Co-ord. num’rs & denom’rs	0.2	1.6	6.6	15.1	20.8	22.7	22.8	22.1
7	Equivalent fractions	0.0	0.1	1.3	5.0	9.1	14.2	19.7	24.7
8	Ordered fractions		0.1	0.3	1.5	5.3	10.1	16.7	9.6
	Total stages 7–8	***0.0***	***0.2***	***1.6***	***6.5***	***14.4***	***24.3***	***36.4***	***34.3***

Appendix B – continued

Percentages of Year 2–9 Students at Framework Stages on Each Domain (Initial and Final) in 2006

	Year	2	3	4	5	6	7	8	9
	Number of students	*5177*	*5161*	*5744*	*5830*	*6206*	*9515*	*8853*	*6740*
	Place Value								
	Initial								
	Not entered	8.3	6.5	6.6	6.0	3.9	4.2	2.5	2.5
0–1	Emergent	19.2	6.5	2.3	0.6	0.3	0.2	0.1	0.2
2	One as a unit	53.0	33.8	14.0	6.8	3.5	1.5	1.3	0.8
3	Five as a unit	3.6	8.2	6.1	3.5	2.5	1.9	0.8	1.9
4	Ten as a counting unit	15.6	42.6	60.3	58.6	47.0	32.1	22.4	11.3
5	Tens in nos. to 1000	0.3	2.3	9.5	19.0	28.0	35.0	37.4	44.1
6	Ts, Hs, Ths in whole nos	0.0	0.2	1.1	4.7	10.8	17.4	21.7	20.5
7	10ths in decimals/orders decs		0.0	0.1	0.8	3.3	5.8	9.7	11.6
8	Decimal conversions				0.1	0.6	2.0	4.1	7.2
	Total stages 7–8	***0.0***	***0.0***	***0.1***	***0.9***	***3.9***	***7.8***	***13.8***	***18.8***
	Final								
	Not entered	9.8	8.2	7.8	7.6	7.0	2.8	2.0	10.9
0–1	Emergent	3.8	1.3	0.5	0.2	0.1	0.1	0.1	0.1
2	One as a unit	26.1	9.6	3.6	1.3	0.7	0.5	0.3	0.1
3	Five as a unit	11.6	9.6	5.1	2.2	1.1	0.9	0.3	0.5
4	Ten as a counting unit	46.0	60.0	56.5	39.9	24.7	15.8	9.1	3.1
5	Tens in nos. to 1000	2.4	10.2	19.8	30.5	31.6	32.9	27.9	26.7
6	Ts, Hs, Ths in whole nos	0.2	1.1	5.5	14.1	22.1	26.9	28.3	24.1
7	10ths in decimals/orders decs	0.1	0.2	0.9	3.5	9.5	12.9	18.4	18.1
8	Decimal conversions			0.1	0.8	3.2	7.2	13.5	16.3
	Total stages 7–8	***0.1***	***0.2***	***1.0***	***4.3***	***12.7***	***20.1***	***31.9***	***34.4***
	Basic Facts								
	Initial								
	Not entered	10.8	7.9	7.6	6.9	5.5	4.5	2.7	2.6
0–1	Emergent	58.1	28.1	9.9	3.4	1.6	0.8	0.5	0.5
2	Addition facts to 5	20.2	27.8	17.5	9.1	5.2	2.4	1.4	0.8
3	Addition facts to 10	4.2	9.4	7.6	5.9	3.7	3.9	2.2	1.1
4	Add'n w. 10s & Doubles	6.3	23.4	38.5	31.8	21.3	14.6	10.5	5.8
5	Addition facts	0.3	2.8	15.5	27.7	30.9	29.0	24.0	19.9
6	Subtr'n & Mult'n facts	0.1	0.4	2.9	12.2	22.7	28.4	31.6	47.5
7	Division facts		0.0	0.5	2.7	8.0	14.1	21.7	21.4
8	Common factors & multiples		0.0	0.0	0.2	1.1	2.2	5.3	0.6
	Total stages 7–8	***0.0***	***0.0***	***0.5***	***2.9***	***9.1***	***16.3***	***27.0***	***22.0***
	Final								
	Not entered	9.3	8.0	7.8	7.6	7.1	2.8	2.5	10.9
0–1	Emergent	20.4	7.1	2.2	0.9	0.4	0.3	0.2	0.1
2	Addition facts to 5	19.6	10.2	4.0	2.0	0.9	0.6	0.4	0.2
3	Addition facts to 10	17.0	11.9	5.8	2.7	1.4	1.4	0.6	0.4
4	Add'n w. 10s & Doubles	29.2	40.4	30.2	16.7	8.6	6.4	4.1	2.0
5	Addition facts	3.7	18.0	33.6	32.0	24.8	21.2	14.7	10.8
6	Subtr'n & Mult'n facts	0.8	3.9	12.6	24.4	29.1	30.2	26.6	38.5
7	Division facts	0.0	0.6	3.5	12.3	22.7	28.2	34.2	35.9
8	Common factors & multiples			0.3	1.4	5.0	8.9	16.5	1.1
	Total stages 7–8	***0.0***	***0.6***	***3.8***	***13.7***	***27.7***	***37.1***	***50.7***	***37.0***

Appendix C (Patterns of Performance and Progress)

Composition of the Year 5–9 Cohort 2002–2006

Year	2002	2003	2004	2005	2006	2002	2003	2004	2005	2006
	Year 5					Year 6				
Number of students	*11316*	*18665*	*9868*	*8353*	*5830*	*11818*	*19338*	*9959*	*8689*	*6206*
Additive Domain										
Total Count from One	4.0	3.4	2.2	1.9	2.1	2.2	2.1	1.0	1.3	1.0
Advanced Counting	22.3	22.3	22.1	20.5	18.2	15.2	15.3	16.2	12.9	10.7
Early Additive P–W	50.2	51.5	52.3	51.0	46.0	46.3	47.0	46.3	45.1	39.1
Adv. Additive P–W	23.5	22.7	23.4	24.0	30.2	36.3	35.7	36.5	33.3	40.0
Total stages 6–7	***23.5***	***22.7***	***23.4***	***26.7***	***33.8***	***36.3***	***35.7***	***36.5***	***40.7***	***49.2***
Multiplicative Domain										
Advanced Counting	25.6	24.7	24.8	24.8	22.2	17.9	16.7	15.3	14.9	12.5
Early Additive P–W	30.6	30.1	30.0	30.1	31.8	27.8	26.5	26.2	25.0	25.6
Adv. Additive P–W	26.1	27.7	29.6	29.3	27.6	31.4	33.9	36.1	35.7	34.0
Adv. Multiplicative P–W	9.1	9.1	8.7	8.2	10.7	18.1	17.9	18.2	16.5	19.9
Total stages 7–8	***9.1***	***9.1***	***8.7***	***9.2***	***11.5***	***18.1***	***17.9***	***18.2***	***20.4***	***22.9***
Proportional Domain										
Early Additive P–W	28.7	29.4	34.8	34.6	33.3	27.0	27.1	32.7	30.4	29.2
Adv. Additive P–W	18.8	19.3	18.9	20.2	21.0	22.9	24.4	25.2	26.5	27.3
Adv. Multiplicative P–W	8.4	8.0	7.4	8.1	10.0	16.9	16.0	14.4	15.6	18.2
Adv. Proportional P–W	0.9	0.9	0.5	0.6	0.5	3.5	3.0	2.2	2.4	2.7
Total stages 7–8	***9.3***	***8.9***	***7.9***	***8.7***	***10.5***	***20.4***	***19.0***	***16.6***	***18.0***	***20.9***
Fractions										
Co–ord. num'rs & denom'rs	18.6	15.9	13.0	14.3	15.1	24.4	21.6	19.1	19.1	20.8
Equivalent fractions	3.0	3.1	4.1	3.6	5.0	6.5	7.3	8.3	8.3	9.1
Orders mixed fractions	1.5	1.6	1.6	1.8	1.5	4.8	4.5	4.5	4.8	5.3
Total stages 7–8	***4.5***	***4.7***	***5.7***	***5.4***	***6.5***	***11.3***	***11.8***	***12.8***	***13.1***	***14.4***

Appendix C – continued

Composition of the Year 5–9 Cohort 2002–2006

Year	2002	2003	2004	2005	2006	2002	2003	2004	2005	2006
	Year 7					**Year 8**				
Number of students	*6418*	*13460*	*8374*	*6348*	*9515*	*5802*	*11796*	*7306*	*5911*	*8853*
Additive Domain										
Total Count from One	2.2	3.4	1.4	0.9	0.9	1.7	3.2	0.8	0.4	0.4
Advanced Counting	13.8	14.6	14.0	10.9	8.8	10.2	10.3	8.3	8.4	6.4
Early Additive P–W	43.5	43.1	43.4	41.4	35.5	37.4	36.9	35.6	32.4	27.8
Adv. Additive P–W	40.4	38.9	41.2	36.3	39.2	50.8	49.5	55.1	39.8	39.9
Total stages 6–7	***40.4***	***38.9***	***41.2***	***46.9***	***54.8***	***50.8***	***49.5***	***55.1***	***58.8***	***65.3***
Multiplicative Domain										
Advanced Counting	14.5	14.4	12.1	11.5	10.4	10.7	9.9	7.3	7.6	6.3
Early Additive P–W	25.1	24.1	24.1	22.9	22.6	20.0	19.7	19.7	18.8	16.9
Adv. Additive P–W	31.7	34.5	38.0	37.6	34.2	32.0	33.8	37.3	35.4	34.4
Adv. Multiplicative P–W	23.9	21.2	22.4	19.6	23.4	33.8	31.8	33.8	25.3	29.1
Total stages 7–8	***23.9***	***21.2***	***22.4***	***25.5***	***30.1***	***33.8***	***31.8***	***33.8***	***36.1***	***41.1***
Proportional Domain										
Early Additive P–W	23.9	25.6	28.8	29.0	26.5	21.6	22.4	23.5	23.1	19.7
Adv. Additive P–W	24.8	25.3	26.8	27.8	25.9	24.3	25.8	29.0	27.5	26.5
Adv. Multiplicative P–W	19.0	16.7	17.9	21.2	24.2	22.6	20.8	23.4	27.2	30.9
Adv. Proportional P–W	6.6	4.9	4.3	3.9	6.1	13.6	11.1	9.3	8.4	11.1
Total stages 7–8	***25.6***	***21.6***	***22.2***	***25.1***	***30.3***	***36.2***	***31.9***	***32.7***	***35.6***	***42.0***
Fractions										
Co–ord. num'rs & denom'rs	26.4	22.8	20.0	20.1	22.7	27.8	23.8	22.5	22.0	22.8
Equivalent fractions	9.9	9.4	12.0	11.8	14.2	12.9	13.6	17.5	16.8	19.7
Orders mixed fractions	9.1	7.3	7.2	7.4	10.1	16.0	13.9	12.7	13.8	16.7
Total stages 7–8	***19.0***	***16.7***	***19.2***	***19.2***	***24.3***	***28.9***	***27.5***	***30.2***	***30.6***	***36.4***

Appendix D (Patterns of Performance and Progress)

Information Used to Calculate Effect Sizes for Adjacent Year Comparisons 2005 and 2006

	2005			Additive Domain				2006			Additive Domain			
Year Level	Younger N	After Mean	Older N	Before Mean	Difference between groups	Pooled SD	Effect size	Younger N	After Mean	Older N	Before Mean	Difference between groups	Pooled SD	Effect size
Overall														
2 with 3	5048	3.54	5719	3.57	–0.029	1.118	–0.03	5177	3.53	5161	3.44	0.091	1.083	0.08
3 with 4	5719	4.32	6966	4.21	0.107	0.950	0.11	5161	4.22	5744	4.24	–0.017	0.958	–0.02
4 with 5	6966	4.77	8353	4.55	0.223	0.884	0.25	5744	4.77	5830	4.67	0.105	0.868	0.12
5 with 6	8353	5.04	8689	4.80	0.241	0.853	0.28	5825	5.14	6205	4.94	0.203	0.871	0.23
6 with 7	8689	5.32	6348	4.95	0.369	0.890	0.41	6205	5.45	9511	5.11	0.342	0.884	0.39
7 with 8	6348	5.44	5911	5.14	0.301	0.896	0.34	9511	5.59	8852	5.34	0.251	0.918	0.27
8 with 9	5911	5.69	4068	5.33	0.356	0.928	0.38	8852	5.83	6730	5.30	0.531	0.927	0.57
Average							**0.35**							**0.37**
European														
2 with 3	3199	3.61	3613	3.67	–0.055	1.073	–0.05	3080	3.66	3086	3.59	0.070	1.042	0.07
3 with 4	3613	4.41	4422	4.31	0.094	0.907	0.10	3086	4.35	3469	4.39	–0.041	0.897	–0.05
4 with 5	4422	4.86	5198	4.65	0.215	0.847	0.25	3469	4.89	3696	4.75	0.143	0.824	0.17
5 with 6	5198	5.13	5476	4.88	0.253	0.829	0.30	3696	5.21	3912	5.01	0.202	0.861	0.24
6 with 7	5476	5.39	4107	5.04	0.346	0.862	0.40	3912	5.54	6247	5.20	0.340	0.883	0.39
7 with 8	4107	5.53	3776	5.24	0.290	0.880	0.33	6247	5.68	5855	5.42	0.259	0.905	0.29
8 with 9	3776	5.78	2686	5.43	0.348	0.906	0.38	5855	5.92	4202	5.41	0.505	0.911	0.55
Average							**0.35**							**0.37**
Māori														
2 with 3	972	3.23	1078	3.20	0.036	1.163	0.03	1230	3.24	1185	3.08	0.153	1.104	0.14
3 with 4	1078	4.01	1292	3.93	0.083	0.972	0.09	1185	3.91	1207	3.84	0.077	0.991	0.08
4 with 5	1292	4.51	1590	4.32	0.190	0.901	0.21	1207	4.43	1078	4.40	0.030	0.900	0.03
5 with 6	1590	4.82	1580	4.56	0.257	0.849	0.30	1078	4.91	1067	4.70	0.212	0.874	0.24
6 with 7	1580	5.10	1363	4.78	0.321	0.879	0.37	1067	5.16	1775	4.86	0.303	0.857	0.35
7 with 8	1363	5.26	1255	4.93	0.330	0.862	0.38	1775	5.32	1683	5.12	0.203	0.883	0.23
8 with 9	1255	5.48	825	5.03	0.449	0.920	0.49	1683	5.56	1308	5.15	0.404	0.914	0.44
Average							**0.38**							**0.32**
Pasifika														
2 with 3	320	3.31	370	3.20	0.106	1.139	0.09	401	3.19	408	3.09	0.096	1.046	0.09
3 with 4	370	4.02	550	3.83	0.193	1.036	0.19	408	3.89	456	3.81	0.081	0.977	0.08
4 with 5	550	4.40	726	4.18	0.218	0.902	0.24	456	4.33	427	4.32	0.013	0.866	0.01
5 with 6	726	4.68	687	4.52	0.162	0.809	0.20	427	4.76	442	4.50	0.266	0.904	0.29
6 with 7	687	4.99	395	4.50	0.486	0.888	0.55	442	5.04	656	4.63	0.410	0.929	0.44
7 with 8	395	4.99	438	4.81	0.184	0.902	0.20	656	5.14	557	4.90	0.239	0.960	0.25
8 with 9	438	5.30	195	4.94	0.358	0.889	0.40	557	5.42	665	4.83	0.590	0.913	0.65
Average							**0.34**							**0.41**

Appendix D – continued

Information Used to Calculate Effect Sizes for Adjacent Year Comparisons 2005 and 2006

	2005		Additive Domain					2006		Additive Domain				
Year Level	Younger N	After Mean	Older N	Before Mean	Difference between groups	Pooled SD	Effect size	Younger N	After Mean	Older N	Before Mean	Difference between groups	Pooled SD	Effect size
Low Decile														
2 with 3	781	3.30	978	3.11	0.193	1.173	0.16	1378	3.18	1438	3.10	0.076	1.096	0.07
3 with 4	978	3.99	1468	3.88	0.102	1.047	0.10	1438	3.90	1318	3.77	0.130	1.005	0.13
4 with 5	1468	4.45	1821	4.23	0.223	0.952	0.23	1318	4.35	1110	4.32	0.027	0.952	0.03
5 with 6	1821	4.75	1835	4.49	0.253	0.883	0.29	1110	4.80	1080	4.54	0.265	0.965	0.27
6 with 7	1835	5.01	761	4.67	0.338	0.957	0.35	1080	5.00	1614	4.77	0.230	0.933	0.25
7 with 8	761	5.20	853	4.91	0.295	0.947	0.31	1614	5.24	1512	5.06	0.176	0.946	0.19
8 with 9	853	5.45	456	5.01	0.432	1.005	0.43	1512	5.49	942	4.96	0.536	0.955	0.56
Average							**0.23**							**0.21**
High Decile														
2 with 3	1980	3.75	2257	3.86	–0.108	1.025	–0.11	1993	3.77	2001	3.69	0.083	1.007	0.08
3 with 4	2257	4.53	2584	4.40	0.128	0.867	0.15	2001	4.47	2470	4.48	–0.013	0.886	–0.01
4 with 5	2584	4.94	3264	4.76	0.179	0.816	0.22	2470	5.01	2587	4.84	0.170	0.784	0.22
5 with 6	3264	5.22	3468	4.97	0.255	0.807	0.32	2587	5.34	2869	5.11	0.223	0.821	0.27
6 with 7	3468	5.52	1949	5.09	0.434	0.841	0.52	2869	5.67	3881	5.33	0.348	0.851	0.41
7 with 8	1949	5.61	1590	5.24	0.368	0.860	0.43	3881	5.81	3456	5.53	0.285	0.902	0.32
8 with 9	1590	5.85	1502	5.53	0.323	0.864	0.37	3456	6.07	1692	5.49	0.578	0.898	0.64
Average							**0.24**							**0.27**

	2005		Multiplicative Domain					2006		Multiplicative Domain				
Year Level	Younger N	After Mean	Older N	Before Mean	Difference between groups	Pooled SD	Effect size	Younger N	After Mean	Older N	Before Mean	Difference between groups	Pooled SD	Effect size
Overall														
2 with 3	748	4.19	3176	3.74	0.451	0.752	0.60	566	4.17	2437	3.75	0.419	0.714	0.59
3 with 4	3176	4.46	5769	4.13	0.330	0.876	0.38	2437	4.47	4630	4.18	0.290	0.840	0.34
4 with 5	5769	4.85	7665	4.53	0.326	0.992	0.33	4630	4.85	5325	4.60	0.247	0.981	0.25
5 with 6	7665	5.22	8288	4.90	0.327	1.061	0.31	5325	5.28	5841	4.98	0.303	1.072	0.28
6 with 7	8288	5.66	6187	5.14	0.514	1.123	0.46	5841	5.73	9217	5.26	0.467	1.118	0.42
7 with 8	6187	5.83	5778	5.42	0.411	1.122	0.37	9217	5.90	8711	5.60	0.308	1.136	0.27
8 with 9	5778	6.12	4054	5.74	0.382	1.133	0.34	8711	6.23	6716	5.68	0.553	1.148	0.48
Average							**0.37**							**0.36**
European														
2 with 3	494	4.18	2119	3.73	0.445	0.734	0.61	378	4.21	1655	3.79	0.419	0.712	0.59
3 with 4	2119	4.50	3764	4.20	0.297	0.873	0.34	1655	4.50	3002	4.25	0.257	0.824	0.31
4 with 5	3764	4.93	4824	4.63	0.309	0.988	0.31	3002	4.92	3419	4.69	0.227	0.968	0.23
5 with 6	4824	5.33	5234	5.00	0.329	1.045	0.31	3419	5.38	3735	5.09	0.289	1.061	0.27
6 with 7	5234	5.77	4024	5.27	0.506	1.103	0.46	3735	5.85	6036	5.39	0.460	1.083	0.42
7 with 8	4024	5.95	3695	5.57	0.378	1.105	0.34	6036	6.03	5772	5.71	0.325	1.114	0.29
8 with 9	3695	6.26	2679	5.88	0.383	1.102	0.35	5772	6.34	4198	5.84	0.508	1.122	0.45
Average							**0.37**							**0.36**

Appendix D – continued

Information Used to Calculate Effect Sizes for Adjacent Year Comparisons 2005 and 2006

	2005	Multiplicative Domain						2006	Multiplicative Domain					
Year Level	Younger N	After Mean	Older N	Before Mean	Difference between groups	Pooled SD	Effect size	Younger N	After Mean	Older N	Before Mean	Difference between groups	Pooled SD	Effect size
Māori														
2 with 3	85	4.18	458	3.63	0.548	0.696	0.79	70	3.96	389	3.61	0.351	0.682	0.51
3 with 4	458	4.24	981	3.90	0.343	0.773	0.44	389	4.25	785	3.91	0.340	0.775	0.44
4 with 5	981	4.54	1422	4.29	0.251	0.884	0.28	785	4.56	925	4.33	0.227	0.904	0.25
5 with 6	1422	4.93	1489	4.58	0.356	1.005	0.35	925	4.97	936	4.64	0.332	0.997	0.33
6 with 7	1489	5.31	1325	4.90	0.412	1.072	0.38	936	5.31	1751	4.95	0.369	1.061	0.35
7 with 8	1325	5.55	1231	5.07	0.483	1.077	0.45	1751	5.55	1655	5.29	0.255	1.086	0.23
8 with 9	1231	5.76	820	5.32	0.444	1.102	0.40	1655	5.88	1302	5.49	0.393	1.084	0.36
Average							**0.40**							**0.32**
Pasifika														
2 with 3	37	3.95	163	3.65	0.296	0.715	0.41	35	3.83	142	3.48	0.350	0.583	0.60
3 with 4	163	4.23	405	3.80	0.432	0.795	0.54	142	4.23	316	3.76	0.463	0.714	0.65
4 with 5	405	4.43	635	4.07	0.357	0.857	0.42	316	4.36	380	4.14	0.216	0.824	0.26
5 with 6	635	4.76	653	4.49	0.271	0.953	0.28	380	4.81	407	4.41	0.403	0.939	0.43
6 with 7	653	5.16	369	4.61	0.551	1.018	0.54	407	5.12	626	4.61	0.514	1.064	0.48
7 with 8	369	5.30	419	4.96	0.334	1.056	0.32	626	5.28	542	5.02	0.261	1.089	0.24
8 with 9	419	5.65	194	5.16	0.489	1.105	0.44	542	5.73	658	4.96	0.769	1.091	0.70
Average							**0.40**							**0.46**
Low Decile														
2 with 3	87	4.03	413	3.64	0.395	0.681	0.58	92	4.15	478	3.55	0.602	0.689	0.87
3 with 4	413	4.31	1090	3.88	0.429	0.810	0.53	478	4.20	768	3.82	0.379	0.738	0.51
4 with 5	1090	4.51	1595	4.24	0.276	0.896	0.31	768	4.48	876	4.20	0.279	0.859	0.32
5 with 6	1595	4.90	1723	4.52	0.383	1.008	0.38	876	4.85	868	4.48	0.368	0.987	0.37
6 with 7	1723	5.25	718	4.79	0.459	1.097	0.42	868	5.15	1617	4.82	0.329	1.063	0.31
7 with 8	718	5.47	824	5.03	0.443	1.099	0.40	1617	5.44	1507	5.20	0.240	1.096	0.22
8 with 9	824	5.79	450	5.29	0.504	1.160	0.43	1507	5.83	923	5.23	0.599	1.112	0.54
Average							**0.44**							**0.45**
High Decile														
2 with 3	330	4.29	1477	3.79	0.502	0.776	0.65	314	4.18	1159	3.81	0.366	0.711	0.52
3 with 4	1477	4.53	2280	4.28	0.248	0.889	0.28	1159	4.57	2221	4.32	0.244	0.843	0.29
4 with 5	2280	5.05	3079	4.76	0.295	1.006	0.29	2221	5.04	2437	4.79	0.250	0.982	0.25
5 with 6	3079	5.44	3332	5.11	0.336	1.060	0.32	2437	5.49	2780	5.20	0.297	1.050	0.28
6 with 7	3332	5.88	1912	5.35	0.535	1.111	0.48	2780	5.97	3811	5.50	0.477	1.086	0.44
7 with 8	1912	6.07	1561	5.59	0.485	1.103	0.44	3811	6.17	3414	5.83	0.341	1.107	0.31
8 with 9	1561	6.35	1501	6.05	0.302	1.090	0.28	3414	6.50	1690	5.97	0.531	1.101	0.48
Average							**0.39**							**0.37**

Appendix D – continued

Information Used to Calculate Effect Sizes for Adjacent Year Comparisons 2005 and 2006

	2005	Proportional Domain						2006	Proportional Domain					
Year Level	Younger N	After Mean	Older N	Before Mean	Difference between groups	Pooled SD	Effect size	Younger N	After Mean	Older N	Before Mean	Difference between groups	Pooled SD	Effect size
Overall														
2 with 3	752	3.28	3126	2.57	0.715	1.184	0.60	573	3.23	2362	2.63	0.593	1.133	0.52
3 with 4	3126	3.75	5733	3.09	0.665	1.351	0.49	2362	3.71	4523	3.24	0.470	1.320	0.36
4 with 5	5733	4.20	7618	3.62	0.575	1.470	0.39	4523	4.28	5075	3.80	0.482	1.449	0.33
5 with 6	7618	4.74	8235	4.11	0.631	1.553	0.41	5075	4.77	5693	4.30	0.531	1.557	0.34
6 with 7	8235	5.22	6140	4.57	0.645	1.559	0.41	5693	5.39	8955	4.76	0.630	1.574	0.40
7 with 8	6140	5.50	5726	5.01	0.487	1.521	0.32	8955	5.65	8517	5.23	0.428	1.544	0.28
8 with 9	5726	5.85	4032	5.54	0.314	1.485	0.21	8517	6.05	6654	5.50	0.545	1.472	0.37
Average							**0.34**							**0.35**
European														
2 with 3	481	3.29	2090	2.63	0.667	1.161	0.57	387	3.27	1587	2.68	0.591	1.125	0.53
3 with 4	2090	3.79	3730	3.19	0.608	1.336	0.45	1587	3.80	2932	3.35	0.447	1.304	0.34
4 with 5	3730	4.31	4800	3.76	0.550	1.458	0.38	2932	4.39	3256	3.96	0.427	1.425	0.30
5 with 6	4800	4.88	5198	4.28	0.593	1.524	0.39	3256	4.97	3628	4.47	0.496	1.526	0.32
6 with 7	5198	5.39	3999	4.79	0.599	1.514	0.40	3628	5.57	5880	4.95	0.618	1.523	0.41
7 with 8	3999	5.70	3674	5.25	0.453	1.463	0.31	5880	5.82	5638	5.42	0.408	1.490	0.27
8 with 9	3674	6.06	2669	5.76	0.303	1.420	0.21	5638	6.20	4159	5.76	0.446	1.397	0.32
Average							**0.33**							**0.33**
Māori														
2 with 3	95	3.31	452	2.37	0.936	1.145	0.82	66	3.08	384	2.42	0.657	1.125	0.58
3 with 4	452	3.42	984	2.77	0.649	1.263	0.51	384	3.34	755	2.91	0.429	1.215	0.35
4 with 5	984	3.78	1404	3.31	0.471	1.377	0.34	755	3.91	875	3.37	0.548	1.367	0.40
5 with 6	1404	4.35	1477	3.69	0.656	1.466	0.45	875	4.38	914	3.78	0.599	1.467	0.41
6 with 7	1477	4.72	1308	4.10	0.621	1.541	0.40	914	4.77	1685	4.24	0.537	1.544	0.35
7 with 8	1308	5.03	1207	4.48	0.548	1.504	0.36	1685	5.20	1617	4.69	0.508	1.552	0.33
8 with 9	1207	5.33	812	4.90	0.430	1.501	0.29	1617	5.56	1287	5.17	0.395	1.474	0.27
Average							**0.38**							**0.34**
Pasifika														
2 with 3	36	2.89	156	2.32	0.568	1.182	0.48	37	2.89	143	2.44	0.451	0.994	0.45
3 with 4	156	3.58	405	2.59	0.993	1.311	0.76	143	3.17	313	2.62	0.545	1.160	0.47
4 with 5	405	3.66	637	2.99	0.674	1.325	0.51	313	3.55	363	3.14	0.409	1.262	0.32
5 with 6	637	4.08	652	3.40	0.677	1.446	0.47	363	4.18	400	3.54	0.644	1.446	0.45
6 with 7	652	4.58	365	3.64	0.938	1.504	0.62	400	4.62	611	3.88	0.738	1.526	0.48
7 with 8	365	4.82	417	4.30	0.522	1.532	0.34	611	4.84	532	4.39	0.448	1.524	0.29
8 with 9	417	5.30	193	4.60	0.699	1.460	0.48	532	5.47	652	4.51	0.960	1.526	0.63
Average							**0.48**							**0.46**

Appendix D – continued

Information Used to Calculate Effect Sizes for Adjacent Year Comparisons 2005 and 2006

	2005	Proportional Domain						2006	Proportional Domain					
Year Level	Younger N	After Mean	Older N	Before Mean	Difference between groups	Pooled SD	Effect size	Younger N	After Mean	Older N	Before Mean	Difference between groups	Pooled SD	Effect size
Low Decile														
2 with 3	91	3.11	397	2.31	0.800	1.160	0.69	85	3.07	467	2.40	0.666	1.098	0.61
3 with 4	397	3.54	1093	2.76	0.776	1.303	0.60	467	3.27	743	2.66	0.611	1.179	0.52
4 with 5	1093	3.79	1575	3.19	0.597	1.377	0.43	743	3.61	782	3.14	0.469	1.254	0.37
5 with 6	1575	4.27	1706	3.60	0.675	1.500	0.45	782	4.19	860	3.49	0.707	1.433	0.49
6 with 7	1706	4.69	710	3.90	0.796	1.503	0.53	860	4.45	1549	4.06	0.388	1.534	0.25
7 with 8	710	5.00	814	4.36	0.637	1.541	0.41	1549	4.96	1458	4.58	0.382	1.550	0.25
8 with 9	814	5.37	445	5.01	0.360	1.543	0.23	1458	5.52	932	4.78	0.741	1.553	0.48
Average							**0.48**							**0.42**
High Decile														
2 with 3	338	3.27	1445	2.68	0.589	1.154	0.51	320	3.25	1090	2.64	0.603	1.158	0.52
3 with 4	1445	3.86	2257	3.31	0.548	1.349	0.41	1090	3.86	2151	3.40	0.453	1.365	0.33
4 with 5	2257	4.50	3071	3.96	0.545	1.466	0.37	2151	4.58	2341	4.05	0.533	1.462	0.36
5 with 6	3071	5.03	3320	4.43	0.601	1.528	0.39	2341	5.14	2690	4.63	0.511	1.527	0.33
6 with 7	3320	5.54	1900	4.93	0.606	1.496	0.40	2690	5.75	3673	5.10	0.647	1.478	0.44
7 with 8	1900	5.85	1565	5.35	0.504	1.435	0.35	3673	6.01	3304	5.60	0.414	1.449	0.29
8 with 9	1565	6.20	1499	5.94	0.261	1.369	0.19	3304	6.42	1680	5.94	0.482	1.298	0.37
Average							**0.38**							**0.38**

Appendix E (Patterns of Performance and Progress)

Percentage of Students Who Progressed to a Higher Stage Relative to Initial Stage (by Ethnicity)

Additive Domain	Year 5			Year 6			Year 7		
	European	Māori	Pasifika	European	Māori	Pasifika	European	Māori	Pasifika
Initial Stage									
Stages 0–3 Count All	*153*	*104*	*45*	*101*	*48*	*31*	*99*	*48*	*35*
Up 1	47.1	40.4	60.0	38.6	41.7	41.9	38.4	41.7	37.1
Up 2	24.2	28.8	11.1	27.7	31.3	25.8	15.2	25.0	20.0
Up 3+	1.3	1.9		5.9		6.5	8.1	4.2	
Total	***72.6***	***71.1***	***71.1***	***72.2***	***73.0***	***74.2***	***61.7***	***70.9***	***57.1***
Stage 4 Advanced Counting	*1132*	*458*	*201*	*836*	*379*	*186*	*1047*	*541*	*221*
Up 1	51.8	49.1	38.8	56.9	53.0	52.7	52.2	51.2	51.1
Up 2+	9.8	6.1	7.0	15.6	7.4	7.5	15.8	10.0	9.5
Total	***61.6***	***55.2***	***45.8***	***72.5***	***60.4***	***60.2***	***68.0***	***61.2***	***60.6***
Stage 5 Early Additive	*1827*	*429*	*150*	*1926*	*466*	*179*	*2886*	*815*	*317*
Up 1	36.6	30.3	26.7	46.3	33.3	38.0	48.4	37.4	40.4
Up 2	1.9	1.2		4.5	3.0	1.1		3.4	1.3
Total	***38.5***	***31.5***	***26.7***	***50.8***	***36.3***	***39.1***	***48.4***	***40.8***	***41.7***
Stage 6 Advanced Additive	*560*	*87*	*31*	*956*	*164*	*45*	*1853*	*338*	*78*
Up 1	15.4	14.9	9.7	26.8	9.8	8.9	33.0	22.8	28.2

Additive Domain	Year 8			Year 9			Overall		
	European	Māori	Pasifika	European	Māori	Pasifika	European	Māori	Pasifika
Initial Stage									
Stages 0–3 Count All	*52*	*23*	*19*	*39*	*21*	*27*	*444*	*244*	*157*
Up 1	40.4	30.4	31.6	33.3	28.6	22.2	41.2	38.9	41.4
Up 2	28.8	17.4	47.4	30.8	33.3	51.9	24.1	27.9	27.4
Up 3+	11.5	4.3		10.3	4.8	11.1	5.9	2.5	3.2
Total	***80.7***	***52.1***	***79.0***	***74.4***	***66.7***	***85.2***	***71.2***	***69.3***	***72.0***
Stage 4 Advanced Counting	*786*	*374*	*156*	*465*	*224*	*186*	*4266*	*1976*	*950*
Up 1	47.1	47.9	54.5	53.1	49.1	46.8	52.2	50.2	48.5
Up 2+	22.4	14.9	12.1	18.1	18.3	18.9	16.0	10.6	10.8
Total	***69.5***	***62.8***	***66.6***	***71.2***	***67.4***	***65.7***	***68.2***	***60.8***	***59.3***
Stage 5 Early Additive	*2265*	*749*	*253*	*1783*	*637*	*319*	*10687*	*3096*	*1218*
Up 1	44.5	42.7	44.3	47.7	42.2	38.9	43.5	38.1	38.8
Up 2	8.3	4.9	2.8	6.6	6.4	8.5	5.5	4.0	3.3
Total	***52.8***	***47.6***	***47.1***	***54.3***	***48.6***	***47.4***	***49.0***	***42.1***	***42.1***
Stage 6 Advanced Additive	*2114*	*444*	*106*	*1527*	*370*	*126*	*7010*	*1403*	*386*
Up 1	42.2	29.7	25.5	33.7	27.3	23.0	33.7	24.2	22.0

Appendix E – continued

Percentage of Students Who Progressed to a Higher Stage Relative to Initial Stage (by Ethnicity)

Multiplicative Domain	Year 5			Year 6			Year 7		
	European	Māori	Pasifika	European	Māori	Pasifika	European	Māori	Pasifika
Initial Stage									
Stages 2–3 Count All	*304*	*151*	*83*	*209*	*95*	*57*	*191*	*110*	*87*
Up 1	48.7	55.6	59.0	42.6	47.4	52.6	40.8	40.9	50.6
Up 2	23.7	21.9	16.9	23.9	21.1	26.3	22.5	28.2	14.9
Up 3	6.9	6.0	6.0	20.1	7.4		17.8	13.6	6.9
Up 4+	0.7	0.7		1.0	1.1		0.5	0.9	
Total	***80.0***	***84.2***	***81.9***	***87.6***	***77.0***	***78.9***	***81.6***	***83.6***	***72.4***
Stage 4 Advanced Counting	*1273*	*440*	*191*	*952*	*369*	*188*	*1122*	*542*	*219*
Up 1	43.3	40.9	40.3	43.9	43.9	39.9	41.7	40.6	42.9
Up 2	18.5	12.7	14.1	27.5	18.7	17.6	23.9	15.9	21.0
Up 3+	2.6	0.7	1.0	4.5	2.2	1.6	3.4	3.1	
Total	***64.4***	***54.3***	***55.4***	***75.9***	***64.8***	***59.1***	***69.0***	***59.6***	***63.9***
Stage 5 Early Additive	*1133*	*225*	*80*	*1255*	*293*	*107*	*1880*	*572*	*194*
Up 1	44.8	44.0	35.0	48.7	44.0	43.9	47.4	48.3	46.9
Up 2+	9.9	4.9	3.8	17.0	9.2	11.2	15.0	8.7	10.3
Total	***54.7***	***48.9***	***38.8***	***65.7***	***53.2***	***55.1***	***62.4***	***57.0***	***57.2***
Stage 6 Advanced Additive	*591*	*97*	*20*	*970*	*143*	*50*	*1924*	*399*	*105*
Up 1	36.7	33.0	25.0	42.3	35.0	36.0	40.7	31.1	33.3
Up 2	2.4			3.1	0.7	4.0	5.8	4.3	2.9
Total	***39.1***	***33.0***	***25.0***	***45.4***	***35.7***	***40.0***	***46.5***	***35.4***	***36.2***
Stage 7 Adv. Multiplicative	*115*	*11*	*6*	*323*	*33*	*5*	*809*	*117*	*20*
Up 1	14.8	9.1	0.0	19.2	18.2	0.0	28.7	17.9	35.0

Multiplicative Domain	Year 8			Year 9			Overall		
	European	Māori	Pasifika	European	Māori	Pasifika	European	Māori	Pasifika
Initial Stage									
Stages 2–3 Count All	*126*	*62*	*31*	*48*	*21*	*27*	*878*	*439*	*285*
Up 1	38.1	37.1	48.4	22.9	47.6	40.7	42.6	47.2	52.3
Up 2	28.6	21.0	25.8	16.7	9.5	25.9	23.8	22.6	20.0
Up 3	17.5	24.2	3.2	31.3	23.8	22.2	15.3	11.6	6.3
Up 4+	0.8		3.2	6.3			1.0	0.7	0.4
Total	***85.0***	***82.3***	***80.6***	***77.2***	***80.9***	***88.8***	***82.7***	***82.1***	***79.0***
Stage 4 Advanced Counting	*742*	*353*	*155*	*466*	*225*	*204*	*4555*	*1929*	*957*
Up 1	36.7	40.8	49.7	36.7	35.1	44.6	41.3	40.7	43.3
Up 2	29.8	22.1	20.0	25.8	25.3	20.1	24.3	17.9	18.6
Up 3+	6.0	4.0	4.5	6.5	4.9	5.9	4.2	2.7	2.5
Total	***72.5***	***66.9***	***74.2***	***69.0***	***65.3***	***70.6***	***69.8***	***61.3***	***64.4***
Stage 5 Early Additive	*1511*	*514*	*179*	*1080*	*419*	*246*	*6859*	*2023*	*806*
Up 1	49.6	45.9	54.2	47.2	51.8	42.7	47.7	47.3	45.7
Up 2+	17.9	12.9	15.1	18.3	12.7	20.3	15.7	9.7	13.3
Total	***67.5***	***58.8***	***69.3***	***65.5***	***64.5***	***63.0***	***63.4***	***57.0***	***59.0***
Stage 6 Advanced Additive	*1981*	*511*	*130*	*1435*	*412*	*132*	*6901*	*1562*	*437*
Up 1	41.6	34.6	34.6	38.6	33.3	34.8	40.4	33.3	34.1
Up 2	8.2	3.7	3.1	8.6	6.3	7.6	6.4	4.0	4.3
Total	***49.8***	***38.3***	***37.7***	***47.2***	***39.6***	***42.4***	***46.8***	***37.3***	***38.4***
Stage 7 Adv. Multiplicative	*1153*	*192*	*43*	*874*	*184*	*45*	*3274*	*537*	*119*
Up 1	36.3	22.4	20.9	30.8	25.5	13.3	30.5	22.0	18.5

Appendix E – continued

Percentage of Students Who Progressed to a Higher Stage Relative to Initial Stage (by Ethnicity)

Proportional Domain	Year 5			Year 6			Year 7		
	European	Māori	Pasifika	European	Māori	Pasifika	European	Māori	Pasifika
Initial Stage									
Stage 1 Unequal sharing	*224*	*127*	*63*	*178*	*88*	*49*	*169*	*101*	*53*
Up 1	60.3	63.8	63.5	41.0	60.2	53.1	45.0	49.5	54.7
Up 2	23.2	23.6	25.4	30.9	18.2	38.8	29.6	28.7	30.2
Up 3+	8.0	3.9	1.6	21.4	6.8	4.1	17.2	13.9	1.9
Total	***91.5***	***91.3***	***90.5***	***93.3***	***85.2***	***96.0***	***91.8***	***92.1***	***86.8***
Stages 2–4 Equal sharing	*1531*	*493*	*221*	*1284*	*453*	*216*	*1544*	*683*	*292*
Up 1	38.9	38.7	31.2	42.4	36.2	36.1	36.7	41.0	40.1
Up 2	15.0	7.1	11.8	22.1	13.9	11.1	19.3	14.6	12.7
Up 3+	3.2	1.4	1.4	6.3	2.4	2.8	7.2	3.5	1.4
Total	***57.1***	***47.2***	***44.4***	***70.8***	***52.5***	***50.0***	***63.2***	***59.1***	***54.2***
Stage 5 Early Additive	*1029*	*199*	*63*	*1164*	*254*	*100*	*1758*	*525*	*173*
Up 1	35.7	30.7	23.8	43.0	35.4	40.0	35.4	38.1	38.7
Up 2	13.2	8.5	3.2	16.7	11.8	8.0	19.1	11.5	8.1
Total	***48.9***	***39.2***	***27.0***	***59.7***	***47.2***	***48.0***	***54.5***	***49.6***	***46.8***
Stage 6 Advanced Additive	*370*	*48*	*14*	*655*	*95*	*29*	*1438*	*278*	*73.0*
Up 1	40.3	33.3	28.6	45.2	32.6	41.4	45.7	38.5	37.0
Up 2	1.9			4.1	5.3		6.4	4.0	2.7
Total	***42.2***	***33.3***	***28.6***	***49.3***	***37.9***	***41.4***	***52.1***	***42.5***	***39.7***
Stage 7 Adv. Multiplicative	*101*	*8*	*2*	*327*	*22*	*5*	*860*	*89*	*20*
Up 1	11.9	0.0	0.0	21.1	4.5	20.0	25.8	18.0	20.0

Proportional Domain	Year 8			Year 9			Overall		
	European	Māori	Pasifika	European	Māori	Pasifika	European	Māori	Pasifika
Initial Stage									
Stage 1 Unequal sharing	*96*	*73*	*35*	*18*	*11*	*18*	*685*	*400*	*218*
Up 1	51.0	43.8	57.1	55.6	36.4	22.2	50.1	55.0	54.6
Up 2	22.9	32.9	22.9	38.9	45.5	44.4	27.2	26.0	30.7
Up 3+	19.8	8.2	11.5		9.1	5.6	15.2	8.3	4.2
Total	***93.7***	***84.9***	***91.5***	***94.5***	***91.0***	***72.2***	***92.5***	***89.3***	***89.5***
Stages 2–4 Equal sharing	*1059*	*497*	*182*	*539*	*283*	*249*	*5957*	*2409*	*1160*
Up 1	32.6	40.0	42.9	44.2	46.6	45.4	38.5	40.1	39.2
Up 2	21.2	17.3	22.5	15.6	14.5	15.7	18.8	13.5	14.4
Up 3+	11.5	5.2	5.5	10.2	6.4	6.8	7.1	3.5	3.4
Total	***65.3***	***62.5***	***70.9***	***70.0***	***67.5***	***67.9***	***64.4***	***57.1***	***57.0***
Stage 5 Early Additive	*1440*	*498*	*174*	*1194*	*491*	*225*	*6585*	*1967*	*735*
Up 1	40.1	36.5	40.8	28.2	23.8	22.2	36.5	33	33.1
Up 2	22.1	17.9	16.1	25.5	17.1	20.0	19.1	14	12.9
Up 3	0.9	0.8	1.1	1.3	1.4	0.4	0.8	0.8	0.7
Total	***63.1***	***55.2***	***58.0***	***55.0***	***42.3***	***42.6***	***56.4***	***47.8***	***46.7***
Stage 6 Advanced Additive	*1490*	*335*	*103*	*737*	*205*	*73*	*4690*	*961*	*292*
Up 1	47.2	38.8	29.1	52.8	48.8	37.0	46.8	40	34.2
Up 2	6.1	3.6	2.9	7.3	4.4	6.8	5.8	3.9	3.4
Total	***53.3***	***42.4***	***32.0***	***60.1***	***53.2***	***43.8***	***52.6***	***43.9***	***37.6***
Stage 7 Adv. Multiplicative	*1297*	*187*	*36*	*1450*	*268*	*86*	*4035*	*574*	*149*
Up 1	32.7	26.2	25.0	21.9	17.9	19.8	25.9	19.9	20.8

Appendix F (Patterns of Performance and Progress)

Percentage of Students Who Progressed to a Higher Stage Relative to Initial Stage (by Decile)

Additive Domain	Year 5			Year 6			Year 7		
	Low	Mid	High	Low	Mid	High	Low	Mid	High
Initial Stage									
Stages 0–3 Count All	*125*	*127*	*68*	*81*	*59*	*46*	*67*	*85*	*44*
Up 1	48.8	45.7	48.5	42.0	45.8	32.6	35.8	40.0	38.6
Up 2	14.4	27.6	30.9	23.5	23.7	41.3	16.4	18.8	20.5
Up 3+	0.8	1.6		2.5	3.4	8.7	1.5	5.9	13.7
Total	***64.0***	***74.9***	***79.4***	***68.0***	***72.9***	***82.6***	***53.7***	***64.7***	***72.8***
Stage 4 Advanced Counting	*520*	*692*	*713*	*427*	*583*	*520*	*527*	*883*	*499*
Up 1	45.8	50.1	53.4	53.4	58.7	54.6	51.8	54.0	49.1
Up 2+	6.9	7.2	11.8	6.3	10.2	20.0	8.5	15.0	16.8
Total	***52.7***	***57.3***	***65.2***	***59.7***	***68.9***	***74.6***	***60.3***	***69.0***	***65.9***
Stage 5 Early Additive	*370*	*967*	*1348*	*431*	*1070*	*1429*	*711*	*1907*	*1747*
Up 1	29.2	30.3	40.8	28.8	40.2	51.4	37.0	39.4	46.2
Up 2	1.4	1.3	2.5	2.3	2.7	5.7	3.7	4.9	6.4
Total	***30.6***	***31.6***	***43.3***	***31.1***	***42.9***	***57.1***	***40.7***	***44.3***	***52.6***
Stage 6 Advanced Additive	*85*	*247*	*442*	*137*	*427*	*786*	*271*	*931*	*1300*
Up 1	8.2	14.2	19.2	15.3	22.7	26.7	22.1	29.0	36.3

Additive Domain	Year 8			Year 9			Overall		
	Low	Mid	High	Low	Mid	High	Low	Mid	High
Initial Stage									
Stages 0–3 Count All	*37*	*39*	*31*	*46*	*43*	*10*	*356*	*353*	*199*
Up 1	40.5	43.6	16.1	17.4	41.9	30.0	39.9	43.6	36.7
Up 2	27.0	23.1	51.6	45.7	34.9	50.0	22.2	25.2	35.2
Up 3+	5.4	7.7	9.7	6.5	11.6		2.5	4.5	6.5
Total	***72.9***	***74.4***	***77.4***	***69.6***	***88.4***	***80.0***	***64.6***	***73.3***	***78.4***
Stage 4 Advanced Counting	*350*	*670*	*361*	*203*	*592*	*152*	*2027*	*3420*	*2245*
Up 1	47.4	46.1	53.7	40.9	53.2	50.0	48.7	52.3	52.6
Up 2+	14.6	18.8	23.9	16.3	21.4	19.7	9.4	14.5	17.3
Total	***62.0***	***64.9***	***77.6***	***57.2***	***74.6***	***69.7***	***58.1***	***66.8***	***69.9***
Stage 5 Early Additive	*653*	*1558*	*1286*	*441*	*1849*	*716*	*2606*	*7351*	*6526*
Up 1	43.2	43.5	45.9	45.6	43.5	47.2	37.5	40.2	46.3
Up 2	4.3	6.2	11.8	7.7	7.4	7.4	4.0	5.0	6.6
Total	***47.5***	***49.7***	***57.7***	***53.3***	***50.9***	***54.6***	***41.5***	***45.2***	***52.9***
Stage 6 Advanced Additive	*403*	*1247*	*1289*	*223*	*1307*	*620*	*1119*	*4159*	*4437*
Up 1	23.6	37.0	49.0	33.2	27.8	42.7	23.0	29.5	37.5

Appendix F – continued

Percentage of Students Who Progressed to a Higher Stage Relative to Initial Stage (by Decile)

Multiplicative Domain	Year 5			Year 6			Year 7		
	Low	Mid	High	Low	Mid	High	Low	Mid	High
Initial Stage									
Stages 2–3 Count All	*169*	*236*	*182*	*114*	*163*	*130*	*146*	*163*	*108*
Up 1	52.7	56.8	47.3	46.5	50.3	36.2	47.9	37.4	41.7
Up 2	20.1	21.2	27.5	24.6	25.8	25.4	19.9	27.0	23.1
Up 3	6.5	3.4	9.3	7.0	10.4	22.3	8.9	17.2	16.7
Up 4+	0.6		1.1	0.9	1.2	0.8	1.4		0.9
Total	***79.9***	***81.4***	***85.2***	***79.0***	***87.7***	***84.7***	***78.1***	***81.6***	***82.4***
Stage 4 Advanced Counting	*451*	*786*	*842*	*386*	*653*	*597*	*538*	*848*	*635*
Up 1	38.4	42.1	45.7	44.0	44.6	41.2	40.5	42.9	40.0
Up 2	13.7	16.8	19.0	16.6	22.4	32.0	16.5	20.5	26.0
Up 3+	1.1	1.7	3.3	1.3	3.8	4.0	2.4	1.9	5.8
Total	***53.2***	***60.6***	***68.0***	***61.9***	***70.8***	***77.2***	***59.4***	***65.3***	***71.8***
Stage 5 Early Additive	*180*	*578*	*829*	*228*	*688*	*986*	*504*	*1257*	*1115*
Up 1	39.4	44.6	46.9	40.4	46.4	51.3	46.6	45.3	49.8
Up 2+	3.3	8.7	10.5	9.2	13.4	17.0	8.5	12.2	18.2
Total	***42.7***	***53.3***	***57.4***	***49.6***	***59.8***	***68.3***	***55.1***	***57.5***	***68.0***
Stage 6 Advanced Additive	*66*	*273*	*475*	*117*	*458*	*755*	*330*	*1086*	*1263*
Up 1	31.8	36.3	36.8	23.1	36.7	47.4	31.2	35.5	45.6
Up 2		2.2	2.1	1.7	4.4	2.5	5.2	5.7	5.8
Total	***31.8***	***38.5***	***38.9***	***24.8***	***41.1***	***49.9***	***36.4***	***41.2***	***51.4***
Stage 7 Adv. Multiplicative	*9*	*53*	*105*	*21*	*144*	*286*	*86*	*373*	*595*
Up 1	11.1	7.5	13.3	4.8	20.8	22.4	25.6	27.9	28.7

Multiplicative Domain	Year 8			Year 9			Overall		
	Low	Mid	High	Low	Mid	High	Low	Mid	High
Initial Stage									
Stages 2–3 Count All	*64*	*104*	*64*	*37*	*51*	*23*	*530*	*717*	*507*
Up 1	42.2	41.3	29.7	40.5	29.4	26.1	47.9	46.7	40.0
Up 2	25.0	20.2	34.4	10.8	29.4	39.1	20.9	24.0	27.4
Up 3	15.6	18.3	17.2	21.6	29.4	17.4	9.4	12.1	15.6
Up 4+	1.6	1.0		5.4			1.3	0.4	0.8
Total	***84.4***	***80.8***	***81.3***	***78.3***	***88.2***	***82.6***	***79.5***	***83.2***	***83.8***
Stage 4 Advanced Counting	*356*	*614*	*353*	*236*	*567*	*143*	*1967*	*3468*	*2570*
Up 1	43.8	38.9	34.8	37.3	39.5	30.8	40.9	41.8	40.9
Up 2	22.2	24.8	34.3	19.9	24.7	32.9	17.3	21.5	26.6
Up 3+	4.8	4.9	7.1	3.8	6.0	9.1	2.5	3.4	5.0
Total	***70.8***	***68.6***	***76.2***	***61.0***	***70.2***	***72.8***	***60.7***	***66.7***	***72.5***
Stage 5 Early Additive	*475*	*1059*	*848*	*274*	*1242*	*381*	*1661*	*4824*	*4159*
Up 1	49.9	49.1	47.5	44.5	46.9	52.5	45.6	46.6	49.4
Up 2+	13.9	14.2	22.8	13.2	16.7	21.8	10.4	13.5	17.8
Total	***63.8***	***63.3***	***70.3***	***57.7***	***63.6***	***74.3***	***56.0***	***60.1***	***67.2***
Stage 6 Advanced Additive	*446*	*1257*	*1177*	*249*	*1302*	*601*	*1208*	*4376*	*4271*
Up 1	37.4	36.4	45.7	39.4	36.1	38.6	34.4	36.1	44.0
Up 2	3.1	7.2	9.2	10.4	7.5	9.8	4.9	6.3	6.3
Total	***40.5***	***43.6***	***54.9***	***49.8***	***43.6***	***48.4***	***39.3***	***42.4***	***50.3***
Stage 7 Adv. Multiplicative	*155*	*628*	*770*	*105*	*688*	*405*	*376*	*1886*	*2161*
Up 1	14.2	32.6	39.7	39.0	26.7	31.9	23.1	27.9	31.7

Appendix F – continued

Percentage of Students Who Progressed to a Higher Stage Relative to Initial Stage (by Decile)

Proportional Domain	Year 5			Year 6			Year 7		
	Low	Mid	High	Low	Mid	High	Low	Mid	High
Initial Stage									
Stage 1 Unequal sharing	*128*	*164*	*169*	*103*	*148*	*118*	*109*	*153*	*86*
Up 1	71.1	54.3	50.3	67.0	39.2	39.8	59.6	45.1	40.7
Up 2	20.3	31.1	23.1	19.4	32.4	33.1	21.1	34.6	30.2
Up 3+	1.6	6.7	11.2	3.9	14.9	16.9	8.3	11.1	17.4
Up 4+			3.6		2.0	5.9		2.0	4.7
Total	***93.0***	***92.1***	***88.2***	***90.3***	***88.5***	***95.7***	***89.0***	***92.8***	***93.0***
Stages 2–4 Equal sharing	*488*	*941*	*1022*	*489*	*813*	*841*	*702*	*1133*	*871*
Up 1	34.4	39.5	38.9	32.9	42.9	43.0	36.2	40.3	36.3
Up 2	9.6	10.6	18.6	11.5	16.4	25.8	13.5	17.8	20.1
Up 3+	1.4	1.7	4.6	1.2	5.0	8.3	2.3	4.7	10.3
Total	***45.4***	***51.8***	***62.1***	***45.6***	***64.3***	***77.1***	***52.0***	***62.8***	***66.7***
Stage 5 Early Additive	*134*	*552*	*753*	*195*	*641*	*893*	*431*	*1154*	*1079*
Up 1	23.9	34.8	36.9	37.9	38.5	44.7	36.2	34.8	37.8
Up 2	6.0	9.6	15.1	7.2	12.5	19.0	6.7	14.2	21.7
Up 3			0.3		0.6	0.7	2.6	0.3	1.2
Total	***29.9***	***44.4***	***52.3***	***45.1***	***51.6***	***64.4***	***45.5***	***49.3***	***60.7***
Stage 6 Advanced Additive	*28*	*171*	*300*	*58*	*317*	*538*	*234*	*847*	*908.0*
Up 1	39.3	38.6	41.7	34.5	42.9	43.5	28.2	42.6	51.0
Up 2		2.9	1.3	5.2	3.2	3.9	9.0	4.4	6.7
Total	***39.3***	***41.5***	***43.0***	***39.7***	***46.1***	***47.4***	***37.2***	***47.0***	***57.7***
Stage 7 Adv. Multiplicative	*4*	*40*	*96*	*14*	*138*	*279*	*64*	*378*	*645*
Up 1	25.0	5.0	12.5	7.1	20.3	21.1	34.4	23.8	25.9

Proportional Domain	Year 8			Year 9			Overall		
	Low	Mid	High	Low	Mid	High	Low	Mid	High
Initial Stage									
Stage 1 Unequal sharing	*54*	*109*	*59*	*26*	*20*	*6*	*420*	*594*	*438*
Up 1	59.3	50.5	33.9	15.4	65.0	16.7	62.1	47.8	42.9
Up 2	27.8	20.2	39.0	53.8	30.0	66.7	23.3	30.3	29.9
Up 3+	5.6	12.8	20.3	3.8	16.7	4.5	10.8	15.3	
Up 4+	1.9	1.8	1.7	3.8			0.5	1.3	4.1
Total	***94.6***	***85.3***	***94.9***	***76.8***	***95.0***	***100.1***	***90.4***	***90.2***	***92.2***
Stages 2–4 Equal sharing	*500*	*860*	*486*	*321*	*685*	*159*	*2500*	*4432*	*3379*
Up 1	41.6	34.4	32.5	34.9	50.1	39.6	36.1	41	38.4
Up 2	18.4	17.7	28.4	14.3	15.0	19.5	13.4	15.6	22.2
Up 3+	5.8	8.9	13.4	6.8	8.6	11.3	3.2	5.5	8.5
Total	***65.8***	***61.0***	***74.3***	***56.0***	***73.7***	***70.4***	***52.7***	***62.1***	***69.1***
Stage 5 Early Additive	*461*	*1005*	*820*	*256*	*1345*	*467*	*1477*	*4697*	*4012*
Up 1	38.6	38.8	38.0	30.1	25.1	28.3	35	33.4	38.1
Up 2	15.4	20.4	24.6	19.5	21.6	27.0	11.6	16.9	21.1
Up 3	1.1	0.5	1.8	1.6	1	1.9	1.4	0.6	1.1
Total	***55.1***	***59.7***	***64.4***	***51.2***	***47.7***	***57.2***	***48.0***	***50.9***	***60.3***
Stage 6 Advanced Additive	*309*	*947*	*878*	*131*	*682*	*289*	*760*	*2964*	*2913*
Up 1	38.2	41.3	51.3	35.1	51	55.4	34.3	43.9	49.2
Up 2	3.2	4.5	8.2	14.5	6.2	6.2	7.0	4.6	6.0
Total	***41.4***	***45.8***	***59.5***	***49.6***	***57.2***	***61.6***	***41.3***	***48.5***	***55.2***
Stage 7 Adv. Multiplicative	*114*	*712*	*869*	*184*	*1122*	*648*	*380*	*2390*	*2537*
Up 1	28.9	29.5	33.8	22.8	19.2	27.0	26.1	22.8	27.9

Appendix G (Patterns of Performance and Progress)

Performance of Students (Initially Below Stage 7, Some Progressed to Stage 7+, Others Did Not) on All Domains

2006	Additive Domain			Multiplicative Domain			Proportional Domain		
	Progress	No Progress	Diff	Progress	No Progress	Diff	Progress	No Progress	Diff
Initially **Finally**	**< St 7** **St 7 A/S**	**St 6 A/S** **< St 7**		**< St 7** **St 7+ M/D**	**St 6 M/D** **< St 7**		**< St 7** **St 7+ P/R**	**St 6 P/R** **< St 7**	
Number of students	*4133*	*6620*		*6447*	*5380*		*6331*	*3578*	
Additive Domain									
Add/Sub Initially									
5 Early Additive	21.9			43.0	57.0		45.6	51.0	
6 Adv. Additive	76.4	100.0		48.5	35.2		44.8	39.0	
7 **Adv. Multiplicative**	**0.0**	**0.0**		**4.2**	**1.5**		**4.9**	**3.0**	
Add/Sub Finally									
5 Early Additive		6.3		6.0	32.7		9.1	30.4	
6 Adv. Additive		93.0		60.6	57.6		61.6	56.3	
7 **Adv. Multiplicative**	**100.0**	**0.0**		**33.1**	**7.2**	**25.9**	**29.0**	**10.8**	**18.2**
Multiplicative Domain									
Mult/Div Initially									
5 Early Additive	14.3	21.8		24.3			28.0	28.8	
6 Adv. Additive	42.3	46.8		69.4	100.0		47.7	52.2	
7 Adv. Multiplicative	34.7	24.8					16.0	12.2	
8 Adv. Proportional	5.1	1.3					1.1	0.2	
Total stages 7–8	***39.8***	***26.1***		***0.0***	***0.0***		***17.1***	***12.4***	
Mult/Div Finally									
5 Early Additive	1.0	8.3			8.6		4.0	14.3	
6 Adv. Additive	13.8	38.7			90.1		31.1	54.4	
7 Adv. Multiplicative	54.3	45.9		88.5			53.4	26.9	
8 Adv. Proportional	30.4	5.4		11.5			11.2	2.2	
Total stages 7–8	***84.7***	***51.3***	***33.4***	***100.0***	***0.0***	***100.0***	***64.6***	***29.1***	***35.5***
Proportional Domain									
Prop/Ratio Initially									
5 Early Additive	2.9	26.5		28.0	38.0		31.2		
6 Adv. Additive	16.0	31.0		36.1	32.6		55.4	100.0	
7 Adv. Multiplicative	52.3	27.1		21.9	12.5				
8 Adv. Proportional	27.9	1.7		1.1	0.3				
Total stages 7–8	***80.2***	***28.8***		***23.0***	***12.8***		***0.0***	***0.0***	
Prop/Ratio Finally									
5 Early Additive	2.5	14.2		6.5	24.3			11.1	
6 Adv. Additive	15.2	29.8		23.9	42.0			85.7	
7 Adv. Multiplicative	55.6	46.3		58.4	26.6		91.4		
8 Adv. Proportional	25.4	5.9		9.8	1.5		8.6		
Total stages 7–8	***81.0***	***52.2***	***28.8***	***68.2***	***28.1***	***40.1***	***100.0***	***0.0***	***100.0***

Appendix G – continued

Performance of Students (Initially Below Stage 7, Some Progressed to Stage 7+, Others Did Not) on All Domains

2006	Additive Domain			Multiplicative Domain			Proportional Domain		
	Progress	No Progress	Diff	Progress	No Progress	Diff	Progress	No Progress	Diff
Initially Finally	< St 7 St 7 A/S	St 6 A/S < St 7		< St 7 St 7+ M/D	St 6 M/D < St 7		< St 7 St 7+ P/R	St 6 P/R < St 7	
Fractions									
Fractions Initially									
5 Orders units fractions	32.0	43.2		43.4	50.5		47.2	49.4	
6 Co-ord. nums/denoms	27.9	23.2		24.9	21.0		25.1	24.2	
7 Equivalent fractions	23.7	12.5		12.3	6.5		10.6	7.5	
8 Order mixed fractions	8.2	3.1		3.1	1.0		2.7	1.8	
Total stages 7–8	***31.9***	***15.6***		***15.4***	***7.5***		***13.3***	***9.3***	
Fractions Finally									
5 Orders units fractions	7.1	26.5		18.3	35.5		17.9	32.3	
6 Co-ord. nums/denoms	18.4	28.6		27.9	33.0		27.5	34.0	
7 Equivalent fractions	36.4	25.2		31.6	16.5		33.3	19.6	
8 Order mixed fractions	32.4	10.7		16.6	4.7		16.0	5.3	
Total stages 7–8	***68.8***	***35.9***	***32.9***	***48.2***	***21.2***	***27.0***	***49.3***	***24.9***	***24.4***
Place Value									
Place Value Initially									
5 10s in nos to 1000	31.7	42.2		41.3	45.7		42.6	44.9	
6 10s, 100s, 1000s whole nos	31.0	27.3		27.2	23.1		27.6	27.5	
7 10th in dec/orders dec to 3 places	18.9	9.2		10.8	5.5		9.4	6.5	
8 Converts decimals to %	7.8	2.7		2.5	0.8		2.4	1.0	
Total stages 7–8	***26.7***	***11.9***		***13.3***	***6.3***		***11.8***	***7.5***	
Place Value Finally									
5 10s in nos to 1000	7.4	24.8		18.2	33.9		19.9	30.4	
6 10s, 100s, 1000s whole nos	21.8	35.3		33.0	35.1		32.8	38.9	
7 10th in dec/orders dec to 3 places	33.4	21.3		27.6	15.5		27.6	16.3	
8 Converts decimals to %	30.5	8.3		14.4	3.9		13.2	4.1	
Total stages 7–8	***63.9***	***29.6***	***34.3***	***42.0***	***19.4***	***22.6***	***40.8***	***20.4***	***20.4***
Basic Facts									
Basic Facts Initially									
5 Add'n & mult'n facts 2, 5, 10	12.9	20.3		21.7	28.4		21.8	23.2	
6 Sub'n & mult'n facts	38.9	37.9		43.8	44.5		43.3	47.0	
7 Division facts	36.6	29.1		23.7	14.1		22.4	18.4	
8 Common factors/ multiples	4.9	2.6		2.0	1.0		2.3	1.4	
Total stages 7–8	***41.5***	***31.7***		***25.7***	***15.1***		***24.7***	***19.8***	
Basic Facts Finally									
5 Add'n & mult'n facts 2, 5, 10	2.0	8.6		5.0	14.0		5.8	11.5	
6 Sub'n & mult'n facts	15.8	29.7		25.2	40.6		24.9	39.5	
7 Division facts	49.1	45.4		50.3	34.2		49.9	37.7	
8 Common factors/ multiples	26.4	8.8		14.2	3.7		13.8	5.1	
Total stages 7–8	***75.5***	***54.2***	***21.3***	***64.5***	***37.9***	***26.6***	***63.7***	***42.8***	***20.9***

Appendix H (Patterns of Performance and Progress)

Performance of Year 5–9 Persistent Counters on Knowledge Domains compared to those at Stage 5 (Final 2006)

	Year 5		Year 6		Year 7		Year 8		Year 9	
Final	St 0–4	St 5	St 0–4	St 5	St 0–4	St 5	St 0–4	St 5	St 0–4	St 5
Number of students	*1182*	*2683*	*725*	*2429*	*930*	*3377*	*606*	*2463*	*403*	*1986*
Additive Domain										
Emergent	0.4		0.1		0.4		0.3		1.0	
1:1 counting	0.4		0.6		0.8		0.7		1.0	
Count from One w. Materials	3.9		2.2		3.2		2.5		2.0	
Count from One w. Imaging	5.3		5.1		5.5		2.5		5.0	
Advanced Counting	89.9		92.0		90.1		94.1		91.1	
Early Additive P–W	0.0	100.0	0.0	100.0	0.0	100.0	0.0	100.0	0.0	100.0
Place Value										
Can't count combined collection	0.9		1.2		0.5		0.8		0.7	0.1
One as a counting unit	5.9	0.2	4.8	0.3	3.9	0.4	3.5	0.3	1.7	0.1
Five as a counting unit	7.1	1.5	5.9	0.9	5.6	0.8	3.5	0.3	6.2	0.5
Ten as a counting unit	62.1	48.6	58.6	36.6	52.0	24.4	45.5	16.8	14.1	5.6
Total	***76.0***	***50.3***	***70.5***	***37.8***	***62.0***	***25.6***	***53.3***	***17.4***	***22.7***	***6.3***
Diffs bet. Counters & EA	***25.7***		***32.7***		***36.4***		***35.9***		***16.4***	
Basic Facts										
No basic facts	4.2	0.1	3.0		3.1		2.5		1.2	0.1
Basic facts up to sums of 5	7.9	0.7	6.8	0.3	5.1	0.3	4.8	0.3	2.0	0.2
Basic facts up to sums of 10	8.1	2.2	8.3	1.0	8.6	1.4	4.6	0.9	3.0	0.5
Add'n with 10s & doubles	35.4	18.3	28.0	11.7	26.2	9.5	22.1	7.4	9.7	3.2
Total	***55.6***	***21.3***	***46.1***	***13.0***	***43.0***	***11.2***	***34.0***	***8.6***	***15.9***	***4.0***
Diffs bet. counters & EA	***34.3***		***33.1***		***31.8***		***25.4***		***11.9***	
Forwards Sequence										
Cannot count to 10	0.3		0.1		0.2		0.2		1.0	
Counts to 10 but no nos after	0.2		0.1		0.6		0.5		0.2	
Gives no. after to 10 not to 20	0.8		0.6		1.0		0.5		1.5	
Gives no. after to 20 not to 100	3.4	0.1	1.9		2.2	0.1	1.8		4.0	0.3
Gives no. after to 100 not to 1000	24.8	5.9	21.8	4.0	17.7	2.8	14.7	2.1	11.2	2.4
Total counters	***29.5***	***6.0***	***24.5***	***4.0***	***21.7***	***2.9***	***17.7***	***2.1***	***17.9***	***2.7***
Diffs bet. counters & EA	***23.5***		***20.5***		***18.8***		***15.6***		***15.2***	
Backwards Sequence										
Cannot count back from 10	0.7		0.3		0.3		0.3		1.2	
Counts from 10 but no nos before	0.4		0.4		0.5		0.8			
Gives no. before to 10 not to 20	1.5		1.2		1.3		1.2		0.5	
Gives no. before to 20 not to 100	4.0		3.2	0.2	2.4	0.1	1.3	0.2	2.2	0.1
Gives no. before to 100 not 1000	28.0	6.6	25.2	4.9	22.5	4.0	16.8	3.2	8.7	1.7
Total counters	***34.6***	***6.6***	***30.3***	***5.1***	***27.0***	***4.1***	***20.4***	***3.4***	***12.6***	***1.8***
Diffs bet. counters & EA	***28.0***		***25.2***		***22.9***		***17.0***		***10.8***	
Numeral Identification										
Cannot identify numerals to 10	0.3		0.1		0.3				0.5	
Identifies numerals to 10	0.4		0.3		0.3		0.5			
Identifies numerals to 20	0.8		0.8		0.4		0.3			
Identifies numerals to 100	2.8		1.8		1.9		1.7			
Identifies numerals to 1000	7.8	1.0	7.2	1.1	5.9	0.3	4.5	0.6	1.0	0.3
Total	***12.1***	***1.0***	***10.2***	***1.1***	***8.8***	***0.3***	***7.0***	***0.6***	***1.5***	***0.3***

Appendix I (Patterns of Performance and Progress)

*Performance of Year 5–9 Students at Each Stage on **Basic Facts** on Stages of Other Domains (Final 2006)*

	Stage on Basic Facts	0–1	2	3	4	5	6	7	8*
	Tasks used to determine stages on the Framework for Basic Facts	Emergent: No recall of addition facts to 5	Addition facts to 5: 2 + 3	Addition facts to 10: 5 + 4 6+□=10	Add'n facts with 10 & Dbs: 6 + 6, 9 + 9, 10 + 4, 7 + 10	Addition facts: 8 + 6, 6 + 9, 8 x 5, 5 x 7	Sub'n & mult'n facts: 17 – 9, 15 – 6, 6 x 7, 8 x 4	Division facts: 56 ÷ 7, 63 ÷ 9	Common factors (81, 72) & multiples (8, 12)
	Number of students	*126*	*280*	*456*	*2612*	*7457*	*11 063*	*10 262*	*2779*
	Additive Domain								
0	Emergent	7.9	0.4	0.2	0.0				
1	1:1 counting	10.3	1.4	0.2	0.1				
2	Count from One w. Materials	32.5	8.9	2.9	0.7	0.0	0.0		
3	Count from One w. Imaging	11.9	12.5	8.6	1.5	0.3	0.1	0.0	
4	Advanced Counting	33.3	57.5	48.7	37.3	16.0	5.5	0.9	0.2
5	Early Additive P–W	4.0	17.9	36.0	51.5	58.5	43.8	14.0	2.2
6	Advanced Additive P–W		1.4	2.9	8.6	23.9	43.1	58.0	31.5
7	Adv. Multiplicative P–W			0.7	0.2	1.2	7.4	27.0	66.1
	Total stages 6–7	***0.0***	***1.4***	***3.6***	***8.8***	***25.1***	***50.5***	***85.0***	***97.6***
	Multiplicative Domain								
n/a	Not entered or applicable	65.9	20.4	12.5	3.7	0.4	0.2	0.1	0.1
2–3	Count from One	11.1	17.5	15.8	7.0	1.5	0.4	0.0	
4	Advanced Counting	19.0	51.1	47.6	44.6	20.6	5.5	1.1	0.1
5	Early Additive P–W	3.2	9.3	18.2	31.9	42.7	24.8	8.4	1.3
6	Advanced Additive P–W		1.8	5.0	11.5	29.5	47.6	35.3	11.7
7	Adv. Multiplicative P–W	0.8		0.9	1.4	4.9	18.5	43.4	45.1
8	Adv. Proportional P–W				0.1	0.3	3.0	11.6	41.7
	Total stages 6–8	***0.8***	***1.8***	***5.9***	***13.0***	***34.7***	***69.1***	***90.3***	***98.5***
	Proportional Domain								
n/a	Not entered or applicable	63.5	22.2	13.6	4.7	1.5	0.7	0.3	0.4
1	Unequal sharing	8.7	7.1	3.7	3.4	1.1	0.3	0.0	
2–4	Equal sharing	20.6	57.9	55.7	50.6	28.2	10.3	2.4	0.5
5	Early Additive P–W	7.1	12.1	21.1	31.7	44.6	41.7	12.3	1.8
6	Advanced Additive P–W		0.7	5.0	7.9	18.1	34.1	28.5	11.2
7	Adv. Multiplicative P–W			0.9	1.7	6.3	20.4	46.4	46.9
8	Adv. Proportional P–W				0.1	0.2	2.5	10.1	39.3
	Total stages 6–8	***0.0***	***0.7***	***5.9***	***9.7***	***24.6***	***57.0***	***85.0***	***97.4***
	Fractions								
n/a	Not entered or applicable	61.9	21.4	11.4	3.7	0.3	0.2	0.1	0.1
2–3	Unit fractions not recognised	14.3	26.1	20.2	12.4	4.5	1.1	0.2	
4	Unit fractions recognised	15.9	33.6	36.6	34.3	21	9.5	2.5	0.6
5	Ordered unit fractions	7.9	18.2	28.1	43.6	56.5	44.3	22.2	4.9
6	Co-ord. num'rs & denom'rs		0.7	3.1	5.4	14.3	29.9	28.2	11.6
7	Equivalent fractions			0.4	0.5	2.9	11.7	31.9	27.3
8	Ordered fractions			0.2	0.1	0.5	3.4	14.9	55.5
	Total stages 6–8	***0.0***	***0.7***	***3.7***	***6.0***	***17.7***	***45.0***	***75.0***	***94.4***

Appendix I – continued

*Performance of Year 5–9 Students at Each Stage on **Basic Facts** on Stages of Other Domains (Final 2006)*

	Stage on Basic Facts	**0–1**	**2**	**3**	**4**	**5**	**6**	**7**	**8***
	Tasks used to determine stages on the Framework for Basic Facts	Emergent: No recall of addition facts to 5	Addition facts to 5: 2 + 3	Addition facts to 10: 5 + 4 6+?=10	Add'n facts with 10 & Dbs: 6 + 6, 9 + 9, 10 + 4, 7 + 10	Addition facts: 8 + 6, 6 + 9, 8 x 5, 5 x 7	Sub'n & mult'n facts: 17 – 9, 15 – 6, 6 x 7, 8 x 4	Division facts: 56 ÷ 7, 63 ÷ 9	Common factors (81, 72) & multiples (8, 12)
	Place Value								
n/e	Not entered	0.8	0.0	0.2	0.3	0.2	0.1		0.1
0–1	Emergent	18.3	1.4	0.7	0.0	0.0	0.0	0.1	
2	One as a unit	41.3	19.3	4.6	1.7	0.3	0.1	0.0	
3	Five as a unit	13.5	9.6	15.4	4.1	1.0	0.4	0.0	
4	Ten as a counting unit	24.6	61.1	60.3	62.7	34.5	12.2	3.1	0.6
5	10s in nos. to 1000	1.6	8.6	15.6	26.5	47.1	40.5	21.5	4.9
6	10s, 100s, 1000s in whole nos			2.9	4.2	14.6	33.3	34.3	15.6
7	10ths in decimals/orders decs			0.4	0.3	2.0	9.9	27.5	28.0
8	Decimal conversions				0.1	0.3	3.6	13.5	50.7
	Total stages 6–8	***0.0***	***0.0***	***3.3***	***4.6***	***16.9***	***46.8***	***75.3***	***94.3***

*Only primary and intermediate students were given the opportunity to do stage 8 basic facts.

Appendix J (Patterns of Performance and Progress)

*Performance of Year 5–9 Students at Each Stage on **Place Value** on Stages of Other Domains (Final 2006)*

	Stage on Place Value	**0–1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
	Tasks used to determine stages on the Framework for Place Value	Emergent	One as a unit	Five at a unit	Ten as a unit	10s in nos. to 1000	10s 100s, 1000s in whole nos	10th in decimals, orders decimals	Decimals con-versions
	Number of students	*35*	*204*	*342*	*6384*	*11144*	*8881*	*4878*	*3221*
	Additive Domain								
0	Emergent	25.7	1.5		0.0			0.0	
1	1:1 Counting	17.1	7.4		0.0				
2	Count from One w. Materials	25.7	23.0	3.8	0.4	0.0			
3	Count from One w. Imaging	5.7	14.2	9.4	1.3	0.2	0.0	0.0	
4	Advanced Counting	20.0	36.8	52.6	29.2	8.2	2.6	0.5	0.1
5	Early Additive P–W	5.7	16.2	30.1	55.5	49.2	28.5	10.3	3.0
6	Advanced Additive P–W		1.0	4.1	13.2	39.1	57.0	51.3	26.8
7	Adv. Multiplicative P–W				0.4	3.2	11.9	37.8	70.1
	Total stages 6–7	***0.0***	***1.0***	***4.1***	***13.6***	***42.3***	***68.9***	***89.1***	***96.9***
	Multiplicative Domain								
n/a	Not entered or applicable	74.2	45.1	11.4	2.3	0.2	0.2	0.2	0.1
2–3	Count from One	11.4	20.1	18.7	4.5	0.6	0.1	0.0	
4	Advanced Counting	8.6	26.0	40.6	35.5	10.1	2.4	0.4	0.0
5	Early Additive P–W	2.9	6.9	21.9	36.3	34.1	15.2	4.2	0.6
6	Advanced Additive P–W	2.9	2.0	7.3	18.8	40.9	47.6	29.3	10.4
7	Adv. Multiplicative P–W				2.5	12.9	31.0	51.3	40.8
8	Adv. Proportional P–W				0.0	1.1	3.5	14.6	48.1
	Total stages 6–8	***2.9***	***2.0***	***7.3***	***21.3***	***54.9***	***82.1***	***95.2***	***99.3***
	Proportional Domain								
n/a	Not entered or applicable	74.3	47.5	12.3	3.3	1.0	0.7	0.3	0.1
1	Unequal sharing	2.9	9.8	4.7	2.5	0.4	0.1		
2–4	Equal sharing	17.1	30.9	58.8	42.5	16.1	5.1	1.0	0.1
5	Early Additive P–W	5.7	11.3	19.0	37.3	39.8	21.6	5.8	1.1
6	Advanced Additive P–W		0.5	5.0	12.3	27.9	37.3	23.4	7.0
7	Adv. Multiplicative P–W			0.3	2.0	14.1	32.4	57.7	45.0
8	Adv. Proportional P–W				0.1	0.8	2.7	11.9	46.9
	Total stages 6–8	***0.0***	***0.5***	***5.3***	***14.4***	***42.8***	***72.4***	***93.0***	***98.9***
	Fractions								
n/a	Not entered or applicable	80.0	47.1	10.8	2.2	0.1	0.2	0.1	0.0
2–3	Unit fractions not recognised	8.6	25.5	30.1	9.5	1.6	0.4	0.0	
4	Unit fractions recognised	8.6	18.1	31.3	32.8	12.9	4.1	0.7	0.2
5	Ordered unit fractions	2.9	8.3	26.3	48.3	54.9	33.3	10.9	2.7
6	Co-ord. num'rs & denom'rs		1.0	0.9	6.6	22.1	37.3	25.8	9.0
7	Equivalent fractions			0.6	0.6	7.0	19.3	41.6	30.8
8	Ordered fractions				0.1	1.2	5.4	20.8	57.3
	Total stages 6–8	***0.0***	***1.0***	***1.5***	***7.3***	***30.3***	***62.0***	***88.2***	***97.1***

Appendix J – continued

Performance of Year 5–9 Students at Each Stage on ***Place Value*** *on Stages of Other Domains (Final 2006)*

	Stage on Place Value	**0–1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
	Tasks used to determine stages on the Framework for Place Value	Emergent	One as a unit	Five at a unit	Ten as a unit	10s in nos. to 1000	10s 100s, 1000s in whole nos	10th in decimals, orders decimals	Decimals con- versions
	Basic Facts								
n/e	Not entered	2.9	0.5	0.3	0.1	0.2	0.4	0.5	0.2
0–1	Emergent	65.7	25.5	5.0	0.5	0.0			
2	Addition facts to 5	11.4	26.5	7.9	2.7	0.2			
3	Addition facts to 10	8.6	10.3	20.5	4.3	0.6	0.1	0.0	
4	Addition w. 10s & doubles	2.9	22.1	31.6	25.7	6.2	1.2	0.2	0.1
5	Addition facts	2.9	10.3	22.5	40.3	31.5	12.2	3.0	0.7
6	Subtr'n & mult'n facts	5.7	4.4	11.7	21.2	40.2	41.5	22.3	12.2
7	Division facts		0.5	0.6	4.9	19.8	39.6	57.9	43.1
8	Common factors & multiples				0.3	1.2	4.9	16.0	43.7
	Total stages 6–8	***5.7***	***4.9***	***12.3***	***26.4***	***61.2***	***86.0***	***96.2***	***99.0***

Appendix K **(The Development of Algebraic Thinking: Results of a Three-year Study)**

Britt Algebraic Thinking Test

Instructions for students

1. This test has four sections, A, B, C, and D. Each section starts on a new page.
2. Look carefully at both the shaded examples at the beginning of each section.

   Then use the method or methods shown in the examples to work out the answers.
3. Do not use a calculator.
4. Write the answers in the space **below** each question.
5. If you cannot see how to use the method to work out a problem, just leave the problem and go on to the next one.

SECTION A (Britt Algebraic Thinking Test)

Jason uses a simple method to work out problems like **27 + 15** and **34 + 19** in his head.

Problem	*Jason's calculation*
27 + 15	30 + 12 = 42
34 + 19	33 + 20 = 53

(1) Show how to use Jason's method to work out **298 + 57**.

(2) Show how to use Jason's method to work out **35.7 + 9.8**.

(3) Use Jason's method to work out what goes in the shaded box in
58 + *n* = 60 + ______ .

(4) Use Jason's method to work out what goes in the shaded box in
9.9 + *k* = 10 + ______ .

(5) Use Jason's method to work out what goes in the shaded box in
***a* + *b* = (*a* + *c*) +** ______ .

SECTION B (Britt Algebraic Thinking Test)

Witi uses a simple method to work out problems like **28 x 5** and **16 x 25** in his head.

Problem	*Witi's calculation*
28 x 5	14 x 10 = 140
16 x 25	4 x 100 = 400

(1) Show how to use Witi's method to work out **286 x 50**.

(2) Show how to use Witi's method to work out **4.8 x 2.5**.

(3) Use Witi's method to work out what goes in the shaded box in **40 x *m* = 10 x** ______ .

(4) Use Witi's method to work out what goes in the shaded box in **6.2 x *p* = 12.4 x** ______ .

(5) Use Witi's method to work out what goes in the shaded box in ***a* x *b* = (*a* ÷ *k*) x** ______ .

SECTION C (Britt Algebraic Thinking Test)

Kate uses a simple method to work out problems like **37 – 18** and **71 – 43** in her head.

Problem	*Kate's calculation*
37 – 18	39 – 20 = 19
71 – 43	68 – 40 = 28

(1) Show how to use Kate's method to work out **182 – 49**.

(2) Show how to use Kate's method to work out **16.1 – 5.2**.

(3) Use Kate's method to work out what goes in the shaded box in
$47 - d = 50 - (\quad)$.

(4) Use Kate's method to work out what goes in the shaded box in
$f - 9.9 = \square - 10$.

(5) Use Kate's method to work out what goes in the shaded box in
$a - b = \square - (b + c)$.

SECTION D (Britt Algebraic Thinking Test)

Kiri uses a simple method to work out problems like **65 ÷ 5** and **300 ÷ 25** in her head.

Problem	*Kiri's calculation*
65 ÷ 5	130 ÷ 10 = 13
300 ÷ 25	1200 ÷ 100 = 12

(1) Show how to use Kiri's method to work out **850 ÷ 50**.

(2) Show how to use Kiri's method to work out **4 ÷ 2.5**.

(3) Use Kiri's method to work out what goes in the shaded box in
$n \div 25 = \square \div 100$.

(4) Use Kiri's method to work out what goes in the shaded box in
$m \div 2.4 = \square \div 24$.

(5) Use Kiri's method to work out what goes in the shaded box in
$a \div b = \square \div (b \times k)$.

MINISTRY OF EDUCATION
Te Tāhuhu o te Mātauranga

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Contracted researchers

F. Ell, N. Hāwera, J. Higgins, K. C. Irwin, A. Tagg, G. Thomas, T. Trinick, J. Ward, J. Young-Loveridge

NZCER researchers

J. Fisher, A. Neill

Ministry of Education researcher

B. Annan

Associated researchers

M. Britt, S. Sharma, B. Sherley, B. Stevenson, M. Taylor, S. Tait-McCutcheon

Foreword

D. Holton

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