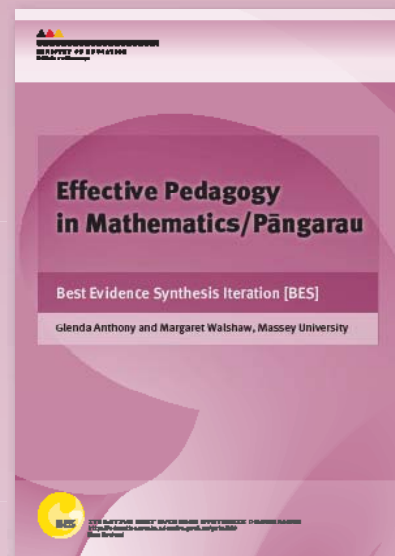


Use a participation framework to support students to discuss their problem-solving strategies

This is one of a series of cases that illustrate the findings of the best evidence syntheses (BESs). Each is designed to support the professional learning of educators, leaders and policy makers.



BES cases: Insight into what works

The best evidence syntheses (BESs) bring together research evidence about ‘what works’ for diverse (all) learners in education. Recent BESs each include a number of cases that describe actual examples of professional practice and then analyse the findings. These cases support educators to grasp the big ideas behind effective practice at the same time as they provide vivid insight into their application.

Building as they do on the work of researchers and educators, the cases are trustworthy resources for professional learning.

Using the BES cases

The BES cases overview provides a brief introduction to each of the cases. It is designed to help you quickly decide which case or cases could be helpful in terms of your particular improvement priorities.

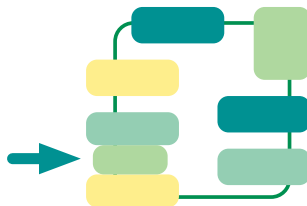
Use the cases with colleagues as catalysts for reflecting on your own professional practice and as starting points for delving into other sources of information, including related sections of the BESs. To request copies of the source studies, use the Research Behind the BES link on the BES website.

The conditions for effective professional learning are described in the Teacher Professional Learning and development BES and condensed into the ten principles found in the associated International Academy of Education summary (Timperley, 2008).

Note that, for the purpose of this series, the cases have been re-titled to more accurately signal their potential usefulness.

Responsiveness to diverse (all) learners

Use the BES cases and the appropriate curriculum documents to design a response that will improve student outcomes



The different BESs consistently find that any educational improvement initiative needs to be responsive to the diverse learners in the specific context. Use the inquiry and knowledge-building cycle tool to design a collaborative approach to improvement that is genuinely responsive to your learners

Use a participation framework to support students to discuss their problem-solving strategies

This case illustrates the importance of positioning students so that they are able to contribute to mathematical discussions – as problem-solvers, solution-reporters, and claim-defenders. Above all, it highlights the importance of ensuring that students who find fractions difficult are treated as competent, with legitimate contributions to make. The teacher in this case carefully supported her students by making use of their prior knowledge and providing them with multiple opportunities to learn.

See also BES Exemplar 1: *Developing communities of mathematical inquiry* and BES Case 29: *Use pedagogical leadership to enable more equitable and effective teaching for all learners.*

Positioning participants as proficient learners

In chapter 4, we found that teachers who produce effective classroom communities seek to develop interrelationships that create spaces for students to develop their mathematical identities. In caring, teachers developed a culture that did not minimise individuals' experiences and contributions within the classroom. Students were trusted with responsibility for themselves and their learning and were provided with opportunities to exercise this responsibility (Angier & Povey, 1999). In CASE 2, the focus is on two low-performing students' experiences in a series of lessons based on equal-sharing fraction tasks. Empson (1999) takes the data from a successful classroom intervention study involving early fraction learning and reanalyses it from a participation perspective, in order to unpack how it is that these two students profited from their classroom experience, "not despite the cognitive or social skill they may have lacked but because of the way their teacher orchestrated their participation in solving and discussing problems" (p. 305).

CASE 2: Low-performing students' participation

(from Empson, 2003)

Mathematics teaching for diverse learners:

- demands an ethic of care;
- demands teacher content knowledge and pedagogical content knowledge and reflecting-in-action;
- involves explicit instructional discourse;
- provides opportunities to explore mathematics through a range of relevant contexts;
- provides tasks that are problematic and have a mathematical focus;
- provides opportunities to resolve cognitive conflicts and problematic reasoning.

This case presents two low-performing students' experiences in a first grade classroom. Explanations for student gains in fraction knowledge are analysed from the perspective of the dynamics of the instructional interactions and their consequences for the students.

Targeted learning outcome

Gains in Patrick and Pho's understanding of fractions.

Learning context

The data are drawn from a larger case study (Empson, 1999) of a US grade 1 class involved in a five-week unit on fractions. The instruction was organised around eliciting and building on children's informal knowledge of equal-sharing situations.

The students

Two students, Patrick and Pho, were assessed in a clinical interview as knowing least about fractions, when compared with their classmates, both at the beginning and at the end of the study. Patrick, from a middle-class family, had been informally identified by school personnel as having trouble focusing on academic tasks. Because he was being withdrawn to participate in an alternative programme, he was present for a total of 9 of the 15 fractions lessons. Pho's family spoke English as a second language. He was present for all 15 lessons.

Pedagogical approach

Assisting students to resolve problematic reasoning is an issue faced by all teachers, especially so in fractions (Thompson & Saldanha, 2003). Empson provides several vignettes that illustrate how the teacher, Ms. K, played a key role in orchestrating the participation of Patrick and Pho in the mathematical practices of the classroom community. The participant frameworks that emerged in her classroom supported Patrick and Pho to participate, when possible, in resolving problematic aspects of their reasoning.

This vignette involves a group situation. Students are solving the problem "There are 2 horses in the field. If they have one bushel with 9 apples in, how many apples would each horse get?" The interaction that follows begins after the first student explained his strategy to the group, including how he split the extra apple in half. When Ms. K called on Pho to report his solution, a predicament emerged with Pho's declaration that he could not give the horses the extra apple:

- 1 Pho: There's one more left, but I can't give them this.
- 2 Ms. K: So what are you going to do with that one left?
- 3 Pho: Uh.
- 4 Ms. K: Anybody got an idea? What can we do with that other one?
- 5 James: I know, I know.
- 6 Ms. K: (To Pho) Do you know what you could do with this one?
- 7 Pho: (Shakes head no)
- 8–9 Ms. K: No. You're not sure. Could anyone share with him what he could maybe do with that other one? James?
- 10 James: Cut it in half.
- 11 Ms. K: Could you cut it in half, Pho?
- 12 Pho: (Inaudible)
- 13 Ms. K: And then how much would each horse get?
- 14 Pho: It still wouldn't work.
- 15 Ms. K: Why won't it work?
- 16 Pho: 'Cause, there still is one more left.
- 17 Ms. K: OK. How much is this horse [pointing to his cubes] going to get?
- 18 Pho: Four.
- 19 Ms. K: How much is this horse [pointing to second horse] going to get?
- 20 Pho: Four.
- 21 Ms. K: James, he says it is still not going to work. Would you please go down—
- 22 Pho: (Interrupting) Because four and four is eight.
- 23–24 James: [To Pho.] You have eight so far. Just cut that (extra apple) in half, and then each one would get eight (sic) and a half.
- 25 Pho: I still can't get it.

In this episode Pho was positioned as a problem solver. Throughout the episode the teacher elicited help from the other children. The distribution of the role of problem solver enabled the other children to act as potential sources for problem-solving ideas. Pho, however, retained the authority to accept or reject the ideas offered based on his understanding of that idea, and in this case he was seen to reject a mathematically legitimate idea proposed by James. While the rest of the group solved a new problem, Ms. K worked with Patrick and Pho to help them arrive at the solution of partitioning the extra apple in half and verbalising how that quantity related to the whole apples (see the following interaction with Patrick).

- 1 Patrick: Hmmm.
- 2 Ms. K: What did the other kids say we could do with this other apple?
- 3 Patrick: Split it in half. But we can't do it.
- 4–5 Ms. K: Well, let's pretend it's [holds up a linking cube] an apple. If it's an apple could we cut it in half? [Patrick agrees.] ... So how many apples is each horse gonna get in this now?
- 6 Patrick: One half.
- 7–8 Ms. K: OK. And they're gonna get these [indicating four linking cubes each] ... How much would they get?
- 9 Patrick: Five.
- 10 Ms. K: How did you figure out five apples?
- 11–12 Patrick: Because if we cut this [extra linking cube] in half they would each get five apples.
- 13–15 Ms. K: Show me. [Patrick counts four single apples each, and on the extra cube, two apples (one for each half).] Is this last piece they're gonna get a whole apple or is that gonna be a half apple?
- 16 Patrick: Half apple.
- 17 Ms. K: Are these [four linking cubes] whole apples or are they half apples?
- 18 Patrick: Whole.
- 19 Ms. K: So how many whole, big apples are they gonna get?
- 20 Patrick: Four.
- 21 Ms. K: OK. And how many half apples?
- 22 Patrick: One. [Patrick writes '4 1' on his paper.]

The interactions were structured so that Patrick and Pho revoiced the key ideas introduced by other children in the earlier exchange. This enabled the two boys to be responsible for evaluating potentially useful mathematical ideas and to begin to make the ideas their own. Throughout the interactions the teacher's use of "what?" and "how?" questions directed the boys' attention to the mathematically critical aspects of the solution—that is, the difference between whole apples and fractional apples as amounts. Physical materials were used as a support for thinking, rather than a literal representation (see Higgins, 2005).

In this and other episodes provided by Empson, we can see that Ms. K oriented Patrick and Pho's participation in instruction towards problem-solving practices and towards taking on an authoritative role. A significant factor in gaining their participation was the acknowledgement and building on the task-based contribution that each boy was able to make. Empson noted that Patrick and Pho had ideas about how to solve almost all of the problems. While these ideas were sometimes partially formed, ambiguously stated, or notationally unsophisticated, they were, with the assistance from Ms. K's scaffolding, able to be treated as part of the pool of ideas for solving the problem. The contributions of Patrick and Pho were accepted or rejected based on mathematical reasons supplied by the learning community.

In the following episode, Pho is positioned as a mathematical authority. This is the first time children are asked to solve an equal-sharing problem involving partition into thirds, a partition that, from a geometric perspective, is harder to make than partitions involving repeated halving (Pothier & Sawada, 1983): "Three children want to share seven candy bars so that everyone gets the same amount. How much would each child get?"

- 1–2 Marie: Because there's—you can't leave one over [i.e., if you make fourths, you will have an extra piece], so if you cut this one [extra cube] in half—

- 3–5 Ms. K: But Marie, look what Pho did [Marie looks at Pho's paper]. Pho, she says you can't split that in three. You think that's right? [Inaudible answer from Pho.] What do you think guys?
- 6–7 Tim: If you split in three, then you would get a half that remains, a half of a candy bar that's still there [Empson noted: he may mean a fourth].
- 8 Ms. K: If you split it in threes?
- 9 Marie: That's what I'm talking about. You still get a quarter left.
- 10 Ms. K: Pho, they said that you can't split it in threes.
- 11 Tim: You can split it in threes but you have a half left.
- 12–13 Ms. K: You'll have a half? Look at how he split it, Tim. Does he have a half left over? [Lev goes over to look at Pho's paper; Tim looks too.]
- 14 Tim: No.
- 15 Ms. K: [To Pho] Do you have a half left over?
- 16 Kaitlin: No.
- 17 Pho: No.
- 18 Tim: That's because of the way you cut it.
- 19 Ms. K: He cut it differently didn't he? Would that work, Tim, or not?
- 20 Tim: It would work.
- 21 Pho: I'll draw it over.
- 22 Ms. K: Would you draw it bigger so people can see.

In this episode Pho was animated by the teacher making a mathematical claim in opposition to another apparently reasonable mathematical claim. In managing the distributed argumentation the teacher created opportunities for Pho to respond directly to Tim and Marie's claim about the impossibility of splitting the bar in three. By relaying their statements to Pho, the teacher effectively scaffolded Pho in this role. In her contribution in lines 12–13, the teacher directed Tim to the part of Pho's diagram illustrating the main claim, thus modelling a move Pho could have made himself. Empson claims that the explicit positioning of the competing ideas assisted the students to resolve the conflicting representations and elevated Pho's solution to the status of a defensible claim of value (lines 20–21). Ultimately, thirds became an acceptable partition in the class. In a later interaction, Patrick was also positioned in the role of author of a partitioning strategy that formed a critical piece in an argument about equivalent fractions.

Learner outcomes

Comparison of the pre- and post-interview results documents Patrick's and Pho's gains in understanding. Throughout the series of lessons Ms. K positioned Patrick and Pho to make contributions to group discussions that enabled them to be animated in identity-enhancing ways—as problem solvers, solution reporters, and claim defenders. These students, although low attainers, were able to successfully engage in problem-solving processes, communicate their thinking, and build complex arguments about mathematical relationships—practices that are essential for learning and doing mathematics (Watson & De Geest, 2005).

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