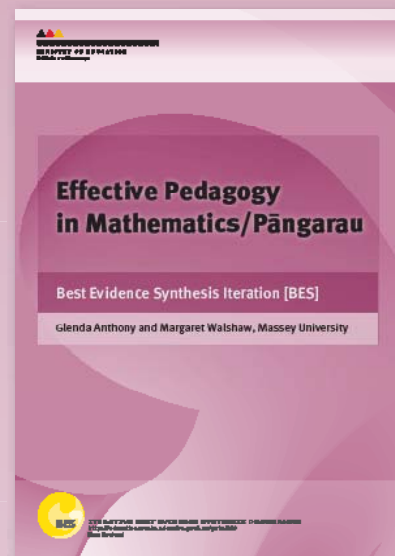# Scaffold learning through the careful selection of tasks and problems

This is one of a series of cases that illustrate the findings of the best evidence syntheses (BESs). Each is designed to support the professional learning of educators, leaders and policy makers.

Effective Pedagogy in Mathematics/Pāngarau

Best Evidence Synthesis Iteration [BES]

Glenda Anthony and Margaret Walshaw, Massey University

BES cases: Insight into what works

The best evidence syntheses (BESs) bring together research evidence about 'what works' for diverse (all) learners in education. Recent BESs each include a number of cases that describe actual examples of professional practice and then analyse the findings. These cases support educators to grasp the big ideas behind effective practice at the same time as they provide vivid insight into their application.

Building as they do on the work of researchers and educators, the cases are trustworthy resources for professional learning.

Using the BES cases

The BES cases overview provides a brief introduction to each of the cases. It is designed to help you quickly decide which case or cases could be helpful in terms of your particular improvement priorities.

Use the cases with colleagues as catalysts for reflecting on your own professional practice and as starting points for delving into other sources of information, including related sections of the BESs. To request copies of the source studies, use the Research Behind the BES link on the BES website.

The conditions for effective professional learning are described in the Teacher Professional Learning and development BES and condensed into the ten principles found in the associated International Academy of Education summary (Timperley, 2008).

Note that, for the purpose of this series, the cases have been re-titled to more accurately signal their potential usefulness.

Responsiveness to diverse (all) learners

Use the BES cases and the appropriate curriculum documents to design a response that will improve student outcomes
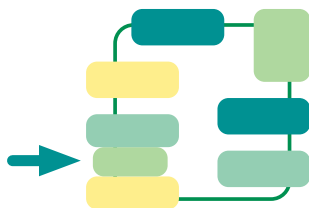
The different BESs consistently find that any educational improvement initiative needs to be responsive to the diverse learners in the specific context. Use the inquiry and knowledge-building cycle tool to design a collaborative approach to improvement that is genuinely responsive to your learners

Scaffold learning through the careful selection of tasks and problems

Teachers need to be careful that their contribution to mathematical conversations does not take from their students valuable opportunities to do some thinking.

This case describes how a secondary school teacher maintained high levels of involvement after noticing that the students were all using variations of the same incorrect strategy to tackle a particular task. The teacher introduced a different but parallel task to demonstrate the shortcomings of the strategy. Once the students saw the issue, the teacher supported them to think about another strategy.

Task engagement and sense making

As we have seen in the previous cases, students' sense making is a key focus of their activities. Providing students with rich tasks within an enquiry-type environment may or may not produce the desired learning activity and outcome (Henningsen & Stein, 1997). In CASE 5, we see how the first assigned task, involving rate calculations, failed to provoke appropriate solution strategies. Exposing teacher indecision as to what to do in such a situation, the case documents a successful way forward.

CASE 5: Calculating Rates

(from Smith, 1998)

Mathematics teaching for diverse learners:

- involves respectful exchange of ideas;
- provides opportunities for children to resolve cognitive conflict;
- involves sequencing of tasks and provision of appropriate challenge;
- utilises appropriate tasks with a mathematical focus—e.g., extreme examples;
- provides opportunities for students to problematise activities based in realistic contexts;
- involves explicit instructional discourse.

In this case, the teacher's first attempt to stimulate students' solution strategies with a rich task only served to affirm existing misconceptions. The teacher needed to reconsider how to challenge these existing misconceptions with a new task.

Targeted learning outcomes

Use of rational equations to solve problems involving rates.

Learning context

Working in a senior secondary remedial mathematics course, the teacher introduced the unit on "rational equations" (p. 750) with the following 'Two Hands Are Better than One' problem:

Darlene and John Edinger were looking for someone to paint their front porch. They received several estimates. The two best estimates came from Michael and Tim. Michael said that he could do the job in 8 hours. Tim told the Edingers that he could complete the job in only 6 hours. Darlene and John wanted the porch painted as quickly as possible, so they decide to hire both men. Approximately how long should it take for Michael and Tim—working together—to complete the job?

Students were required to solve this problem in groups, justifying their solutions to each other. The teacher walked around the room, monitoring their progress.

Student activity

Although the reasoning processes differed in appearance, the group solutions were all based on the misconception that the two painters would work at a rate determined by $^{average\ time}/_2$. For example:

Group A

Michael = 8 hrs (2 people = $^1/_2 \times 8 = 4$); Tim = 6 hrs (2 people = $^1/_2 \times 6 = 3$). So, both together = $(3 + 4)\ 2 = ^7/_2 = 3.5$ hrs.

Group D

8 + 6 = 14, 2 people = 7. The Edingers need the job done as quickly as possible, so they hired both boys. So you take the average of both and divide that number by 2 people. You get the time it will take 2 people to do the job, 3.5 hours.

Teacher reflections

The teacher listened in on a group discussion in which Sally put forward the following argument: "Well I know that the average is seven, and somehow you have to do this, since you are having both of them … I know that seven is not the answer. But we have to find their time together, and I think we need to know the average to get it." The teacher reflected to himself:

I hadn't anticipated this approach! I don't really see how the average could be useful. Should I say something now? Should I let them continue to pursue this conjecture? I'm really not prepared to address this misconception at the moment. This is not what I was expecting. Everyone seems to agree with Sally. Isn't anyone going to question her conjecture? … It looks like the whole group is buying into this idea … Should I be the one to question it? … What a mess!

A new problem

[After some consideration] the teacher constructed a new problem for which the 'average time ÷ 2' method was clearly not a sensible approach:

Suppose that Michael could complete the job in 10 hours and that Tim could complete the job in 2 hours. How long would it take the two men working together to complete the job?

Student activity

Groups initially applied the $^{average\ time}/_2$ algorithm to the new problem. But in some groups, the student reflections on the answer caused some questioning of this approach:

Kerri:	I got three.
Hannah:	I did too.
Lisa:	Wait, you guys, this answer can't be right. It only takes Tim two hours to do it alone.
Tabitha:	[attempting to explain this seemingly impossible answer] Maybe Michael slows Tim down—that could happen. When I work with someone who is real slow, it happens to me.
Lisa:	No. Three is right. We did something wrong.
Kerri:	[agreeing with Tabitha] Maybe Tim did slow down. Why don't the Edingers just hire Tim? Michael is too slow.

Teacher support

When the teacher returned to the whole-class discussion the students were asking good questions and were ready to move forward. They collectively confirmed their suspicion that the $^{average\ time}/_2$ algorithm was not appropriate. Realising that the students needed further guidance if they were to proceed, the teacher offered the following suggestion: "Consider each worker's rate per hour". The groups returned to the problem and successfully found the exact solution, $t = 1^2/_3$ hrs, for $1/_2 + 1/_{10} = 1/_t$ hrs [or $1/_2 + 1/_{10} = 1$].

Quality pedagogy

Factors that facilitated students' sense making with these rate problems included:

- encouragement for students to explain their solutions and develop their own sense of accuracy;
- use of teacher questions to elicit explanations and guide students toward persuasive justification of their solutions;
- a focus on conceptual rather than procedural content. The teacher-provided information focused students on the measurable attribute of the objects in the problem (rate per hour) and the relationships among quantities;

- use of extreme examples;
- respect of student ideas and a recognition that errors can be a useful starting point for effective discussion.

References

Henningsen, M., & Stein, M. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education, 28*(5), 524-549.

Smith, C. (1998). A discourse on discourse: Wrestling with teaching rational equations. *The Mathematics Teacher, 91*(9), 749-753.